To What Extent Can a WhatsApp API-Based Course Develop Primary Teachers' Mathematical Knowledge for Teaching?

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By OLICO Mathematics Education

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Executive Summary

In this report, we discuss the initial iteration of a mathematical knowledge for teaching (MKT) course for primary school mathematics teachers delivered largely remotely via the WhatsApp application programming interface (API) and supplemented by limited in-person sessions. The course is aimed at supporting teachers who are implementing a weekly after-school mathematics program focused on basic concepts in numbers and operations to grade 4 students in South Africa. This design-based research study probed teachers' experiences with the course and the impact of the course on their MKT. These results, alongside the designers' experiences, reflecting both on content and the affordances and constraints of the WhatsApp API, are used to suggest improvements for the next iteration of the course.

Between January and June 2024, the first 14 weeks of this teacher course was piloted and researched. A baseline test of teachers' MKT, as well as a survey of their attitudes toward mathematics and teaching mathematics, was conducted at the initial in-person training in January. After this one-day in-person training session, teachers received weekly lessons via WhatsApp. These lessons consisted of videos and linked questions with automatic feedback. Teachers were provided with checkpoint tests to administer to their students at the end of each four-week cycle. Teachers created one-minute "talking hand" videos demonstrating how they would explain the answer to one of the questions on this test to their students. These videos, along with teachers' reflections on their students' performance, were uploaded to the WhatsApp course and included in the research analysis.

Seventy-one teachers granted us permission to use the data from the surveys, tests, online responses, and videos they created in this research project. In addition, five teachers agreed to take part in two in-depth interviews at the beginning and toward the end of the course, which were used to supplement and deepen the data analysis.

We summarize the results for the research under three headings below:

Teachers' Participation in and Perceptions of the Course

Initial participation in the course was low, and some teachers' were confused about how to access the course. Participation improved after we ran detailed onboarding sessions and streamlined the demands on teachers. This pointed to the importance of keeping the design simple and consistent from week to week, as well as including the onboarding process during the initial in-person training in future iterations of the course.

Despite the improved participation, we still did not get universal uptake by all the teachers even though the course was directly aligned to helping them prepare for the lessons they were about to teach. This both raised concerns about the culture of preparation among the teachers and suggested that it might be worth considering gamification elements to incentivize teachers. It also underscored the importance of creating a dashboard to enable easy access to lesson completion data for teachers and their managers.

Teachers' perceptions of the course were positive, and the vast majority of participants indicated that the course helped them understand mathematics better, showed them new ways to teach math, and helped them explain mathematical ideas in a better way.

Usefulness of the Various Aspects of the Course

In the survey responses, teachers deemed all aspects of the course (the facilitator guide, videos, questions on WhatsApp, and the creation of their own videos) as helpful.

The videos and facilitator guide contain the explicit instruction for the course. There were two further aspects of the course—the questions embedded in WhatsApp and the videos we asked teachers to create—that we as the designers reflected on the usefulness of.

The responses to the questions embedded in WhatsApp were not used to track teacher improvement because they were simple single-number answers or multiple-choice questions—and thus "cheating" would be easy. The questions fell broadly into three of the domains of MKT identified by Ball et al. (2008): common content knowledge (CCK), specialized content knowledge (SCK), and pedagogical content knowledge (PCK). Despite our initial concerns that the limited and simple question types available to us that could be automatically marked in WhatsApp would render the questions of limited use, an analysis of the teachers' responses to the questions indicated otherwise. The CCK questions provided some teachers who got the question wrong an opportunity to be made aware that they were doing a question incorrectly before working on similar questions with their students in class. The SCK questions afforded us an opportunity to hone in on particular misconceptions we had observed among teachers when they were working with models of situations, and the PCK questions provided an opportunity to reinforce the key pedagogical messages of the course.

The one-minute "talking hand" videos that teachers uploaded at the end of every four-week cycle were useful for gaining insight into teachers' thinking and for us to be able to assess the quality of their explanations. However, they were time-consuming to watch and analyze, and feedback was provided at a general level rather than to each teacher individually. A challenge we face in developing the course further is to find a mechanism to provide individual feedback to teachers so that they improve the coherence and clarity of their explanations.

Teacher Performance

Teachers showed a statistically significant improvement between the pre- and post-tests in terms of their basic number fluency and their MKT. Although this is a promising initial result from a pilot, the post-test outcomes indicate that there is still room for improvement, particularly with regard to MKT. Detailed analysis of the questions in the MKT portion of the test, supplemented by the analysis of the brief videos that teachers produced to explain particular concepts, indicated the following:

- gaps in teachers' awareness of working with models of situations, and more familiarity with the calculations needed to produce answers
- a focus on providing the steps to do a calculation, with little or no attention paid to the rationale for those steps
- difficulties with coherence and logic in explanations

- issues linked to language—further investigation is needed to determine the extent to which the medium of instruction (which was not the teachers' home language) played a role and the extent to which mathematical language challenges impacted teachers' performance
- difficulties in working abstractly

Amendments to the initial training, videos, and WhatsApp questions will be made to better address these issues in the next version of the course.

1. Introduction to the Study

A model that has gained traction in South Africa as a means for changing teachers' and children's engagement with mathematics involves after-school math clubs in primary schools (Bowie et al., 2022; Graven et al., 2022). In the clubs, primary school students work with games and activities focused on core number sense. OLICO Mathematics Education, building on the work of the SA Numeracy Chair Project at Rhodes University, began running math clubs in a single township community in 2017. The popularity of this model has led to substantial growth: OLICO currently supports clubs across five provinces involving more than 20,000 students and run by over 200 teachers and 650 youth facilitators on youth employment schemes. OLICO provides detailed facilitator guides to support the leading of clubs—with explicit outlines on how to run the activities, games, and worksheets of each weekly club session—and monitors the learning outcomes.

While learning outcomes are on an upward trajectory, our experiences in this work mirror those of the South African research base: that significant gaps and limitations in primary teachers' mathematical knowledge linked to the content they are teaching (Venkat & Spaull, 2015) represents a "binding constraint" for the possibilities of improving learning outcomes more substantially (Van Der Berg et al., 2016). The need to address this constraint provided the key rationale for this study.

Traditional in-service teacher development programs, in the longitudinal model required to address significant gaps in mathematical knowledge, are either too expensive to implement at scale or not realistic in a context where some facilitators are in remote areas and technology and internet access is limited. Given these constraints, OLICO has been developing and piloting an online, interactive course for our math club teachers and facilitators using the WhatsApp application programming interface (API) powered by turn.io, supplemented with a small number of in-person sessions. The WhatsApp API offers some features that previously required a full-scale learning management system, as well as the opportunity for direct two-way communication. Given the wide-scale use of WhatsApp across South Africa, this is an exciting technology tool with minimal barriers to access. While members of the team have experience with developing courses for primary teachers' mathematics knowledge for teaching (MKT), the novel WhatsApp API mechanism for delivery focused attention on critical content packaging in ways that are important to study within an educational design-based research process, where the focus is on developing our "knowledge about which actions under what circumstances will lead to which kind of intended consequences" (Bakker, 2018, p. 47).

Within this hybrid WhatsApp-linked model, the objectives of this study were to investigate the possibilities for improving primary teachers' knowledge of early number concepts in ways that the research base suggests are important for teaching. For design purposes, we use Ball et al.'s (2008) model of MKT, where developing mathematical knowledge includes the core mathematical knowledge needed to complete tasks for oneself, the specialized knowledge required by teachers to make concepts understandable to others, and the awareness of progressions of ideas that are important for teaching. Our specific research questions were as follows:

1) What is the impact of a hybrid model of development—which encompasses limited in-person training, explicit detailed material, and a WhatsApp course—on math club facilitators' MKT?

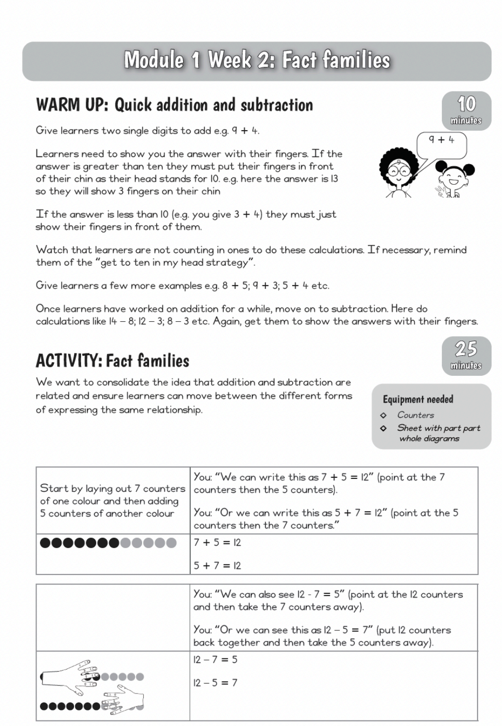
2) What are some of the core challenges and tensions in implementing this model?
3) What adaptations might be necessary to make future iterations of this model more effective?

The lessons learned in this research project have implications more broadly for technology-assisted options for developing the MKT base of South African teachers regarding early grade number concepts.

2. Description of the Project

While the OLICO MKT model will be used to support teachers and youth facilitators running math clubs in locations across the country, we used the short time frame for this research to focus on the efficacy of the model in improving teachers' MKT in the context of one contained project within the broader math clubs program. This project employs teachers to implement the after-school clubs and involves 15 schools spread across four provinces, with 105 teachers working with 2,475 students. The program targets students in grade 4; however, with most South African students in grade 4 two years behind their expected grade level (Spaull & Kotze, 2015) and with further backlogs caused by COVID-19 school shutdowns, the material they work with is drawn from grade 2 and 3 levels. The students attend one 90-minute after-school math session each week, which involves a structured set of activities focused on number work (warm-up, class activity, game, worksheets) led by the teacher. Teachers have a detailed facilitator guide that spells out the set of activities for each week. Excerpts from the module 1 week 2 session are illustrated in Figures 1a and b.

Figure 1a.
Extract from facilitator guide showing warm-up and start of class activity


Module 1 Week 2: Fact families

WARM UP: Quick addition and subtraction

10 minutes

Give learners two single digits to add e.g. 9 + 4.

Learners need to show you the answer with their fingers. If the answer is greater than ten they must put their fingers in front of their chin as their head stands for 10. e.g. here the answer is 13 so they will show 3 fingers on their chin

If the answer is less than 10 (e.g. you give 3 + 4) they must just show their fingers in front of them.

Watch that learners are not counting in ones to do these calculations. If necessary, remind them of the "get to ten in my head strategy".

Give learners a few more examples e.g. 8 + 5; 9 + 3; 5 + 4 etc.

Once learners have worked on addition for a while, move on to subtraction. Here do calculations like 14 – 8; 12 – 3; 8 – 3 etc. Again, get them to show the answers with their fingers.

ACTIVITY: Fact families

25 minutes

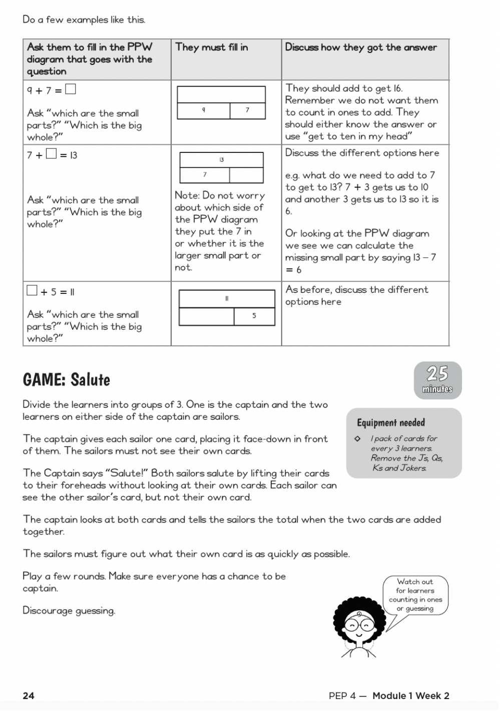
We want to consolidate the idea that addition and subtraction are related and ensure learners can move between the different forms of expressing the same relationship.

Equipment needed
- Counters
- Sheet with part part whole diagrams

Start by laying out 7 counters of one colour and then adding 5 counters of another colour	You: "We can write this as 7 + 5 = 12" (point at the 7 counters then the 5 counters). You: "Or we can write this as 5 + 7 = 12" (point at the 5 counters then the 7 counters."
	7 + 5 = 12 5 + 7 = 12

	You: "We can also see 12 - 7 = 5" (point at the 12 counters and then take the 7 counters away). You: "Or we can see this as 12 – 5 = 7" (put 12 counters back together and then take the 5 counters away).
	12 – 7 = 5 12 – 5 = 7

Figure 1b.
Extract from facilitator guide showing more of class activity and description of the game


Do a few examples like this.

Ask them to fill in the PPW diagram that goes with the question	They must fill in	Discuss how they got the answer
9 + 7 = □ Ask "which are the small parts?" "Which is the big whole?"	9 | 7	They should add to get 16. Remember we do not want them to count in ones to add. They should either know the answer or use "get to ten in my head"
7 + □ = 13 Ask "which are the small parts?" "Which is the big whole?"	13; 7 | Note: Do not worry about which side of the PPW diagram they put the 7 in or whether it is the larger small part or not.	Discuss the different options here e.g. what do we need to add to 7 to get to 13? 7 + 3 gets us to 10 and another 3 gets us to 13 so it is 6. Or looking at the PPW diagram we see we can calculate the missing small part by saying 13 – 7 = 6
□ + 5 = 11 Ask "which are the small parts?" "Which is the big whole?"	11; | 5	As before, discuss the different options here

GAME: Salute

25 minutes

Divide the learners into groups of 3. One is the captain and the two learners on either side of the captain are sailors.

The captain gives each sailor one card, placing it face-down in front of them. The sailors must not see their own cards.

The Captain says "Salute!" Both sailors salute by lifting their cards to their foreheads without looking at their own cards. Each sailor can see the other sailor's card, but not their own card.

The captain looks at both cards and tells the sailors the total when the two cards are added together.

The sailors must figure out what their own card is as quickly as possible.

Play a few rounds. Make sure everyone has a chance to be captain.

Discourage guessing.

Equipment needed
- 1 pack of cards for every 3 learners. Remove the Js, Qs, Ks and Jokers.

24 PEP 4 — Module 1 Week 2

Teachers also attend a full-day in-person training session at the beginning and middle of each year. This training session is based on the teacher guide and aimed at providing teachers with hands-on experience doing and teaching the activities and games they will be leading with their students, as well as discussion of the mathematical and pedagogical principles that underlie them.

In 2024, we supplemented the support given to teachers in the program by adding the WhatsApp course that is the focus of this research. This course was delivered to the teachers weekly. The course was directly linked to the student program. Each week, the teachers were sent a series of two or three short videos aimed at preparing them to deliver the lesson for the following week. These videos supplemented the facilitator guide by providing a more dynamic view of the week's activities. In addition, woven into the exposition of the activities was a discussion of the mathematical ideas that underpin the week's work, along with key pedagogical considerations. The videos were followed by a set of five questions linked to the video material, which teachers answered via WhatsApp and were given immediate feedback on, as illustrated in Figures 2a–c.

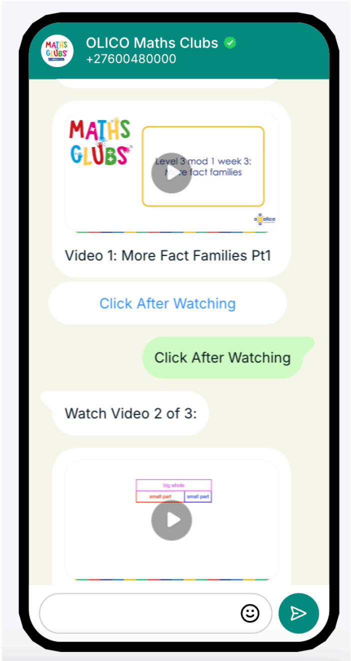


Figure 2a. WhatsApp course video

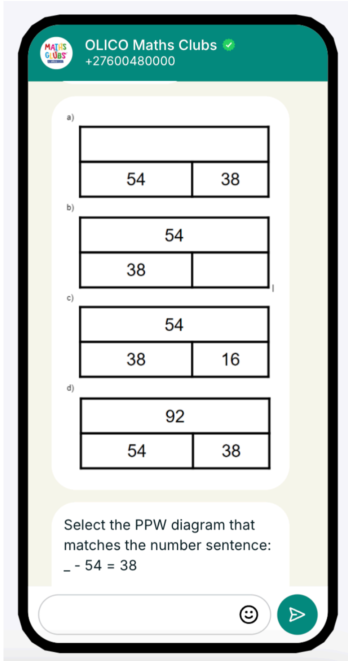


Figure 2b. WhatsApp course question

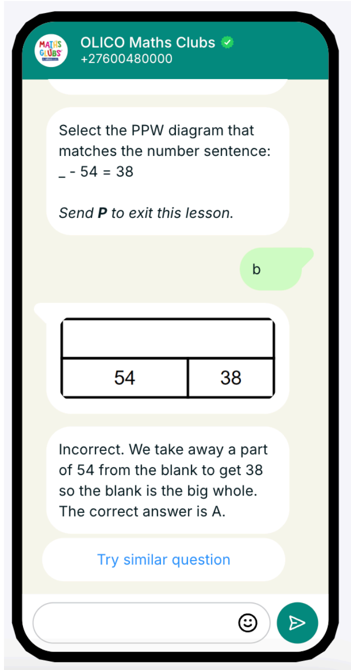


Figure 2c. WhatsApp course feedback on answer to question

At the end of each four-week cycle (which we have termed a "module"), teachers were provided with a checkpoint test to administer to their students. They then uploaded a summary of their results and some reflections on these via WhatsApp. They also uploaded a one-minute "talking hand" video explaining one of the key ideas of the module (see Section 4 for more detail). This research project followed the experience of the teachers during the first half of the course (between January and June 2024).

3. Description of Methods

Design-based research lends itself well to scenarios where the researchers are often also designers and implementers. McKenny and Reeves (2013) describe educational design-based research as the integration of research and development to create educational interventions. They argue that an initial design is based on an analysis of the needs and context. The revision of the design is informed by field investigations using a range of quantitative and qualitative research strategies. The findings of this study—alongside the experiences of designers (reflecting both on content and the affordances and constrains of the WhatsApp API) and trainers, and an evaluation of materials—will feed into further iterations of the MKT content and model of delivery.

In order for this research to inform the revision of the facilitator support materials, we collected data about the impact of the course on all teachers and then took a more in-depth look at the experience of five selected teachers.

i) *The course's impact on all teachers*
In order to look at changes in the teachers' MKT throughout the duration of the intervention, we administered a pre-test at the initial in-person training and a post-test at the mid-year in-person training[1] to all teachers in the course. At the initial in-person training, all teachers were surveyed about their attitudes toward mathematics and mathematics teaching, and at the mid-year in-person training all teachers were surveyed about their experiences with, and attitudes toward, the WhatsApp teacher course.

Data from teachers' answers to the WhatsApp questions were extracted from the app to provide information about teachers' levels of engagement with the online course. Teachers' voice notes and videos submitted as part of the online course were analyzed to provide deeper insight into teachers' MKT.

ii) *In-depth look at the experience of a smaller group of teachers*
In order to get deeper insight into teachers' experience with the course, we selected a group of five teachers for further qualitative work. We selected the teachers so that we had one from each of the five geographical clusters in the program and so that we had one high-performing, one low-performing, and three medium-performing teachers based on their pre-test results. These teachers were visited at their schools early in the year and again toward the end of the intervention. During these visits, the teachers were interviewed about their experience with the course and participated in task-based interviews to probe their MKT. The voice notes and videos submitted by these teachers during the WhatsApp course were analyzed in greater depth alongside the interview data.

This research project was granted ethics approval by the University of Witwatersrand Human Research Ethics Committee (protocol number H24/01/36). All teachers participating in the course were invited to participate in the study, and we received ethics permission for the use of data for research purposes from 71 of the teachers.

In the section below, we have chosen to present certain pieces of data together and immediately follow that presentation with a discussion of the implications for refinement of the course. Although this differs from the traditional format of a research report, in our writing and reflection it provided the

[1] This in-person training session was in preparation for the second set of four modules to be run in the second half of 2024 and thus did not form part of this research. However, we used the fact that the teachers were brought together for this training to administer the post-test.

clearest way to present the evaluation and reflection phase of the design cycle, as described by McKenney and Reeves (2013). Thus, the Results and Discussion section is divided into subsections that describe both the data and our reflections on the data that lead to a particular insight or refinement.

4. Results and Discussion

4.1 Initial Take-Up of the Course

The initial take-up of the course was very disappointing. By the end of February, when all teachers would have taught the lesson for module 1 week 1, only 4% of the teachers had completed the teacher course for that lesson. We thus set up meetings with the teachers who managed the after-school academy at each of the schools. From the discussions in these meetings, we learned that many of these managers and their teachers were confused about how to access the course and, in particular, had conflated the Two Minute Tango App (which we had signed teachers up for to improve their own basic number fluency) and the teacher course on WhatsApp and thus were unsure how to access either of them. We also learned that many of the teachers were complaining about what they termed the excessive demands on their time that the weekly 30-minute WhatsApp lesson (our maximum estimated time for the weekly videos and questions) placed on them. This feedback led to some immediate changes to the course and flagged some considerations for future iterations of the course.

Although we had believed that the Two Minute Tango App would be useful for building teachers' own number fluency, we found that introducing two different technology components simultaneously caused confusion; as a result, we dropped the requirement for teachers to engage with the Two Minute Tango App. For future iterations of the WhatsApp course, given that concerns have been expressed about South African teachers' early number fluencies (Porteus, 2023), it might be worthwhile considering whether a number fluency component could be built into the course. However, this would need to be balanced with the perceived load of the course, discussed further below.

Despite teachers' familiarity with WhatsApp, it was clear that some teachers initially found it difficult to navigate the interactions with a bot rather than a human in this medium. This necessitated one of the OLICO staff members doing onboarding sessions with the groups of teachers at each school. This was time-consuming but effective, and thus, on reflection, we recognized the need to incorporate this onboarding into the initial in-person teacher training.

Teachers' complaints about a lack of time to do the course surprised us. We had assumed (and in fact had been told) that teachers prepared for delivering their weekly lessons. The WhatsApp course was directly aligned to their lessons so that teachers would feel the immediate benefit of being helped to prepare more easily for their lessons, while at the same time allowing us to include core elements of MKT. Teachers' initial reluctance to take on the course suggested that either they did not have a culture of preparing for lessons—meaning that this requirement was indeed an extra burden—or they did not perceive the WhatsApp course as being an efficient way of doing this preparation.

We thus made the following changes to the course:
We changed the way we referred to the course from "The WhatsApp Teachers' Course" to "Prep in Your Pocket" to underscore the notion that the course was aimed to help them with preparation for their classes rather than being an add-on course to learn about mathematics teaching. We also dramatically reduced the amount of work we required from the teachers. We had initially planned a pre- and post-test for each module, ten interactive questions for each week's lesson, and a requirement for teachers to create and upload short videos weekly. We reduced this by cutting out the pre- and post-tests for each module, reducing the number of weekly interactive questions to five, and replacing the weekly video uploads with an end-of-module activity. For this end-of-module activity, we asked teachers to administer a checkpoint test to their students and asked them to give feedback via WhatsApp on their students' progress in terms of key goals of the course (e.g., progression from counting in ones to calculating or fluent recall). We also asked them to upload a voice note reflecting on their students' performance, and we asked them to upload a brief (one-minute maximum) video showing how they would explain one of the questions from the checkpoint to their students. These checkpoints and the accompanying tasks have proved very useful (see Section 4.7 below), and thus we intend to retain them as a central aspect of the course going forward.

4.2 Participation in the Course

Substantial effort was put into getting teachers to join the course. Many teachers indicated that they prepared as a group or with another teacher, with one teacher then uploading task responses. Thus, while Figure 3 shows the percentage of teachers who completed each lesson, we are aware that actual participation levels are higher than these. However, participation levels are still lower than ideal for the aims of the MKT program. Table 1 indicates the percentage of teachers who completed a few, some, or most of the lessons.

Figure 3. Percentage of teachers who completed each lesson

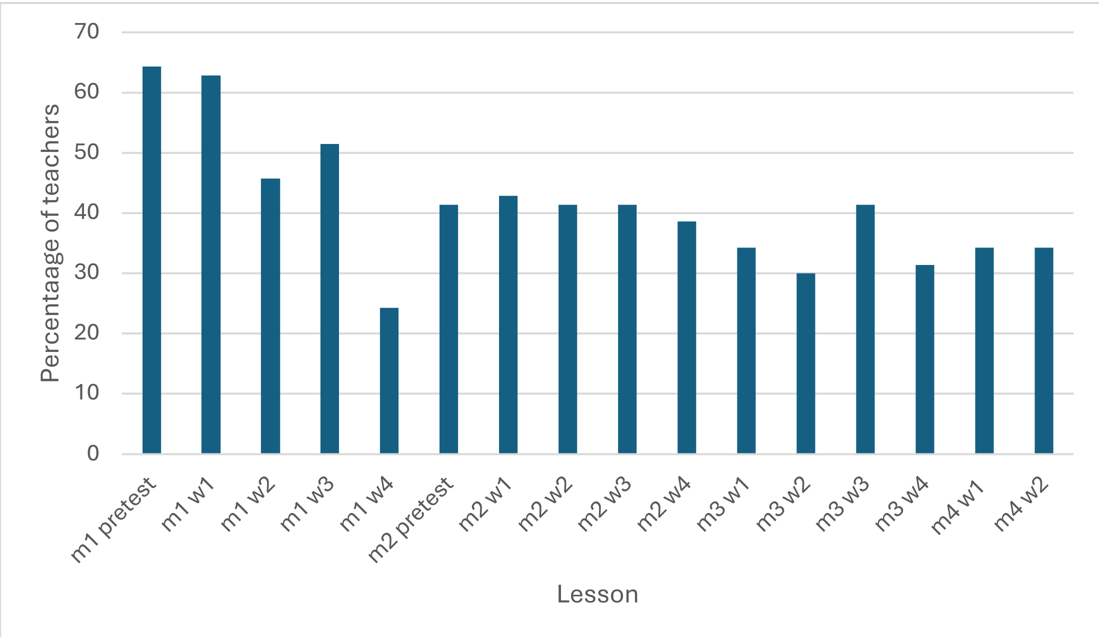


Table 1. Number of lessons completed

Number of lessons completed	0–4 lessons	5–9 lessons	10–13 lessons	All 14 lessons
Percentage of teachers	49%	28%	19%	4%

In future iterations of the course, we need to find ways to increase participation. A clear onboarding process during the initial training—where we can give teachers direct experience in preparing together as a team and yet at the same time have each teacher record their own answers in the WhatsApp course—is clearly one part of the solution. In addition, there is a need to explore feasible "carrot and stick" approaches to increasing participation. For example, gamification has been shown to have some positive effects in terms of engagement in education courses (Majuri et al., 2018; Manzano-Leon et al., 2021), though with some caveats around potential unintended negative consequences (Toda et al., 2018). Similarly, providing a simple dashboard-style report on teachers' completion of lessons to the academy managers would allow for the managers to be able to monitor the engagement of their teachers. Because turn.io is not set up as a student management system, this kind of reporting is not immediately available and would require development.

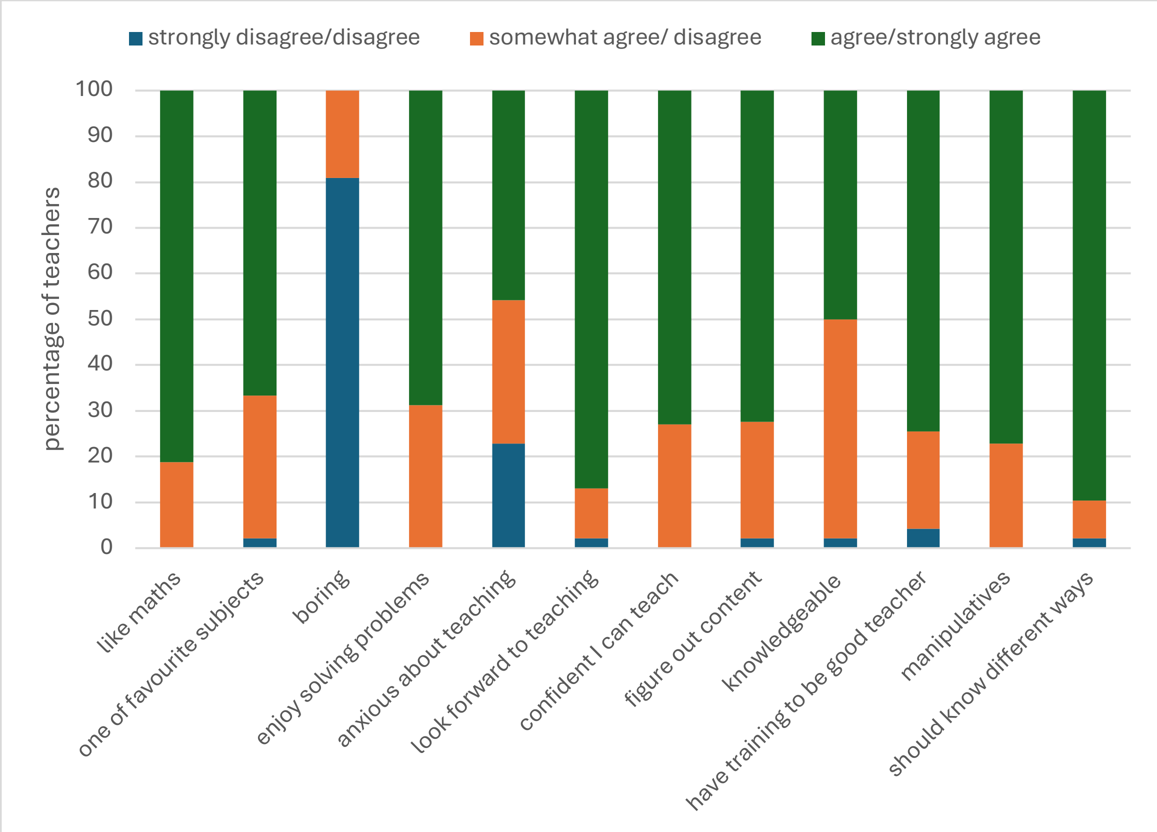
4.3 Pre-Course Survey of Attitudes Toward Mathematics

At the initial training session, teachers completed a survey on their attitudes toward mathematics and teaching mathematics. At this session, 48 of the 71 teachers were present; their responses are summarized in Table 2. The survey consisted of 12 Likert-style questions.

Table 2. Average score (0 = strongly disagree, 1 = disagree, 2 = somewhat disagree, 3 = somewhat agree, 4 = agree, 5 = strongly agree) of teachers (n = 48)

Statement	**Average score**
I like mathematics	4.15
Mathematics is one of my favorite subjects	3.88
I think mathematics is boring	0.72
I enjoy solving mathematics problems	4.02
I am anxious about teaching mathematics	3
I look forward to teaching mathematics	4.24
I am confident in my ability to be a good mathematics teacher	4.06
I can figure out primary school math content even if I don't know it already	3.74
I am knowledgeable in mathematics	3.37
I have the training to be a good mathematics teacher	3.94
I know how to use manipulatives (e.g., counters, ten frames) to help students understand mathematics	4.02
A mathematics teacher should be able to solve a problem in my different ways	4.44

Figure 4. Percentage of teachers who strongly agreed; somewhat agreed or disagreed; strongly disagreed with each of the questions (see Table 2 for full version of questions)

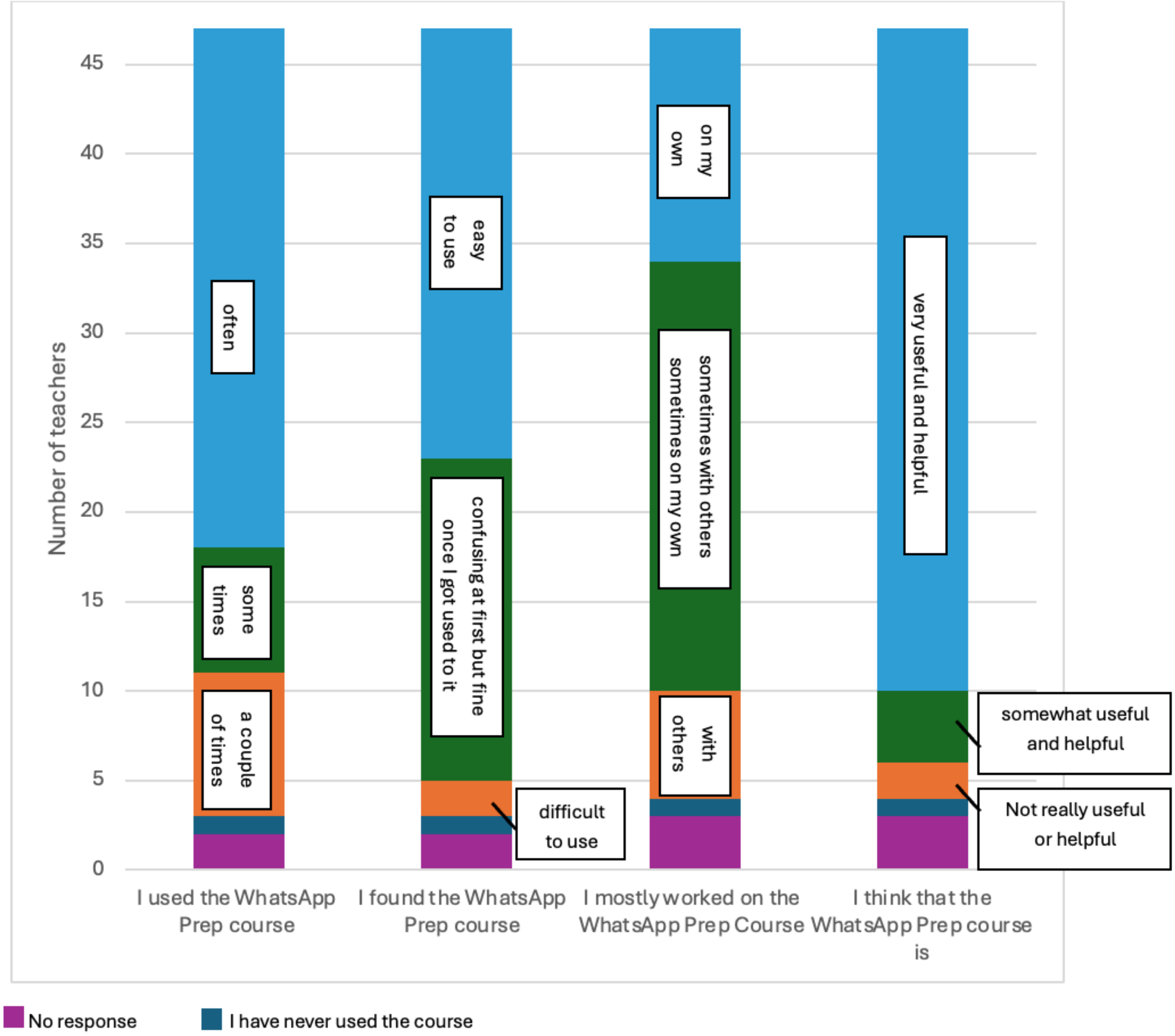


The majority of teachers reported that they liked mathematics and even considered it to be one of their favorite subjects. The teachers also expressed enjoyment in solving mathematics problems and indicated that they could figure out primary mathematics content, even if they didn't know it already. This picture of teachers who are confident about their mathematical abilities is tempered by their response to the statement "I am knowledgeable in mathematics," where approximately half were unable to agree or strongly agree with that statement. The majority of teachers reported looking forward to teaching mathematics and having the training and confidence to do so. However, this again is tempered by the fact that almost half of the teachers indicated that they were anxious about teaching mathematics. Thus, although the initial survey suggests that the teachers feel positive about mathematics and confident in their ability to teach it, the two "tempering" responses suggest that there is a need for some caution in interpreting this. Although the teachers expressed confidence in using manipulatives and were strongly in agreement that teachers should know how to solve a problem in many different ways, this is at odds with the findings of teachers' difficulty using representations and of their strongly calculations-based approaches that emerged through our analysis of their performance on the pre- and post-tests and in the videos they uploaded. These are discussed further below.

4.4 Teachers' Perceptions of the Course

The post-course survey consisted of 16 Likert-style questions that asked teachers about the extent to which they had used the course, their perceptions of the course, and the usefulness of various aspects of the course. The survey also provided a space for teachers to write open-ended comments about what they feel could be improved about the course. Forty-seven of the teachers completed the survey; their responses are shown in Figures 5, 6, and 7.

Figure 5. Teachers' self-reported usage and views on the course (n = 47)



These responses align with the reported completion of rates of lessons download from turn.io in that 29 teachers reported using the course often and a similar number said that they worked together with others. The majority of teachers (even those who did not use the course often) reported finding the course very useful. Most teachers found the course either easy to use or confusing at first, but then fine once they got used to it. This was confirmed by the experience of OLICO staff, who found that they needed to provide initial guidance to get teachers onto the course and familiar with the menu, but also found that after this initial guidance most teachers were able to work independently with the course.

Figure 6. Teachers' perceptions of the course (n = 47)

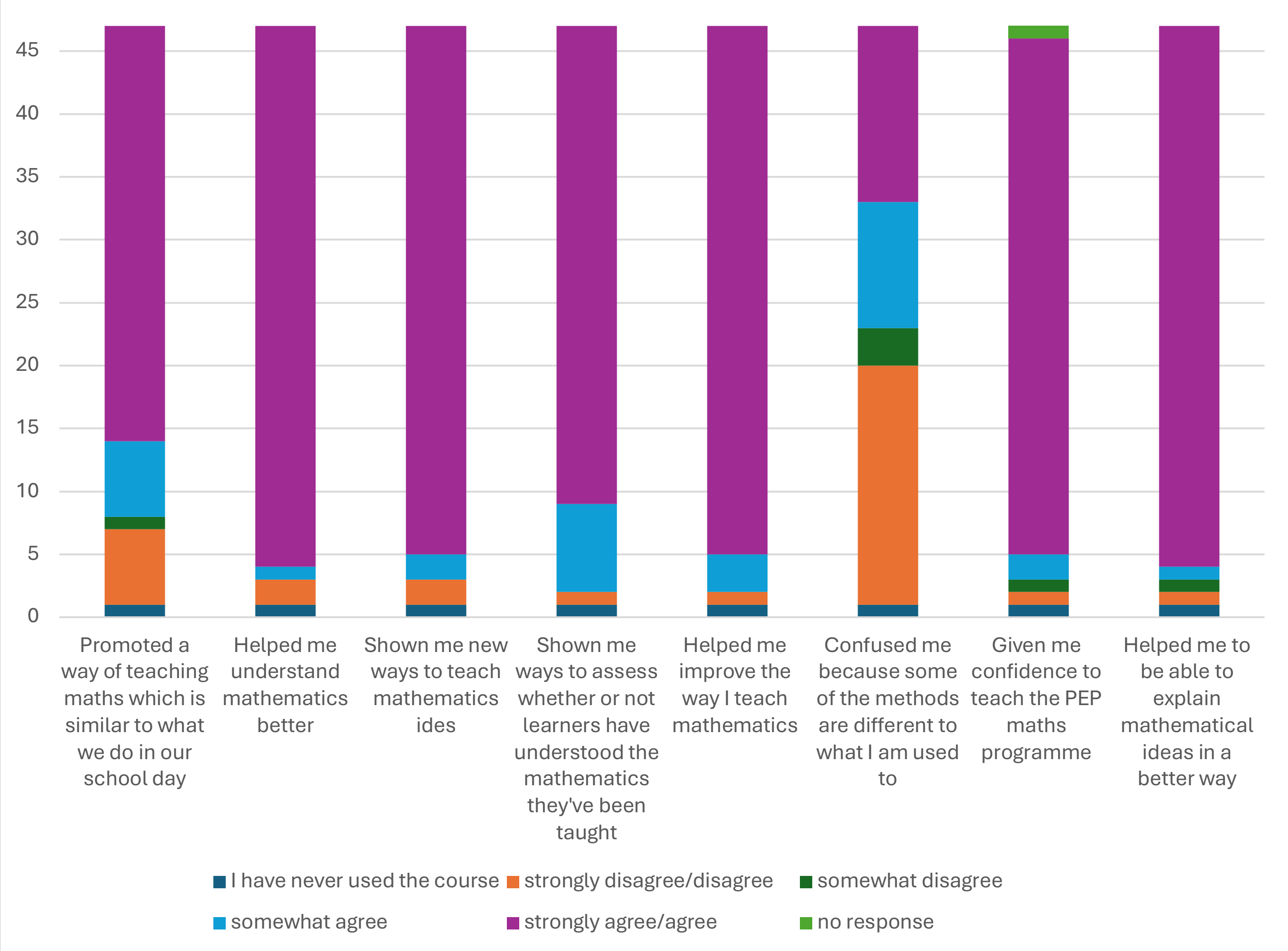


Figure 6 indicates that teachers felt strongly that the course helped them understand mathematics better and both improved the way they teach mathematics and showed them new ways to teach mathematical ideas. They also felt that the course had increased their confidence to teach on the program, helped them explain mathematical ideas better, and improved their ability to assess whether their students are understanding the mathematics they have been taught. There was a more mixed picture in terms of whether the course methods aligned with what they were used to and the way in which mathematics is taught during the school day.

Figure 7. Teachers' rating of the helpfulness of each aspect of the course (n = 47)

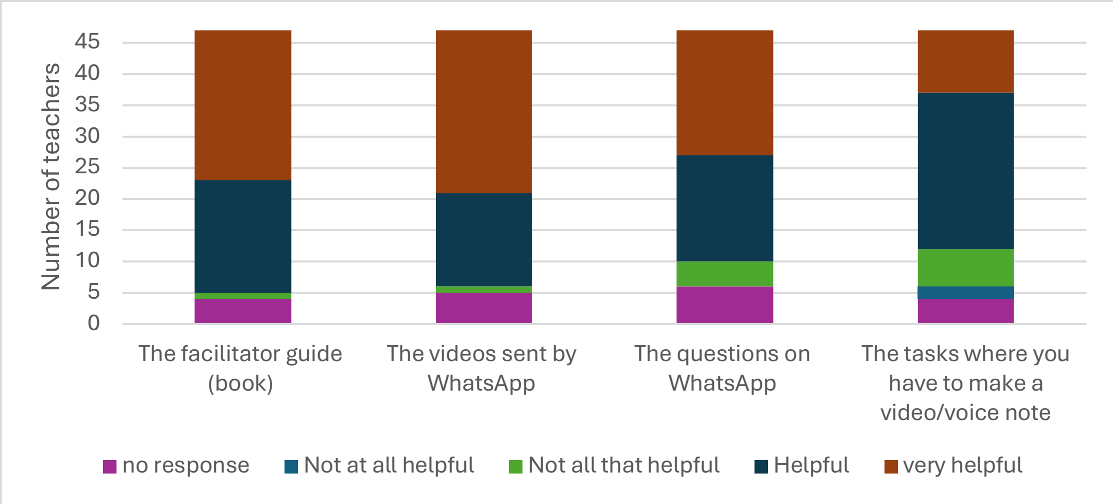


Figure 7 indicates that more than half of the teachers rated both the facilitator guide and videos as very helpful, and most of the rest considered them helpful. We were particularly pleased that most of the teachers also rated the questions on WhatsApp as either helpful or very helpful and rated having to make the video and voice notes as helpful.

4.5 Pre- and Post-Tests

Over the course of the research project, there were a number of changes to the staffing of the after-school program over which we had no control. Thus, some of these 71 teachers were not present at the initial training when the teachers wrote the pre-test, and others had resigned from the program by the time the post-test was written, leaving us with matched pre- and post-test data for only 43 teachers, which we report on below. Three of these teachers arrived late to the session and missed writing the fluency section of the test.

Given that the pre- and post-tests were administered six months apart, we used the same test in both cases because we felt it unlikely that the teachers would have direct recall of the questions. Table 3 shows the overall results of the pre-and post-tests. Each test consisted of a fluency section and an MKT section. In the fluency section, teachers were given two minutes to complete a set of 15 calculations. The calculations were linked with the intention that a teacher who was able to see the relationships would be able to answer the questions quickly and easily. For example, a set of four questions such as 35 + 87 = ___; 87 + ___ = 122; 87 + ___ = 121; and 122 - 35 = ____ was given one after the other. Meanwhile, the MKT section of the test consisted of seven questions.

Table 3. Teachers' performance on the pre- and post-tests

	Pre-test average	Pre-test SD	Post-test average	Post-test SD	Significant p<0.01
Fluency (max score 15) n = 40	8.65	3.82	10.68	2.81	yes
Written MKT test (max score 19) n = 43	6.49	3.04	9.09	3.20	yes

The results show statistically significant improvements in fluency and MKT outcomes across the six-month period. This is a promising initial result from the pilot, even though the post-test outcomes continue to indicate that there is still some way to go to bring MKT in particular to the levels needed for good-quality teaching. Table 4 shows the percentage of teachers who got each of the questions from the MKT section of the test correct.

Table 4. Teachers' performance on MKT items in the pre- and post-tests (n = 43)

		Pre-test	Post-test
Question 1	Match number sentences to part-part-whole diagram	5%	2%
Question 2	Match story problem to part-part-whole diagram	51%	65%
Question 3	Missing tens values in addition calculation	58%	67%
Question 4	Understand the hundred chart	5%	23%
Question 5a	Express expanded horizontal form of two-digit minus two-digit calculation	9%	33%
Question 5b	Show completion of subtraction calculation on number line	21%	44%
Question 6	Explain how to solve missing minuend problem to students	0%	5%
Question 7a	Write a story problem to match a given subtraction calculation using the word "more"	16%	28%
Question 7b	Write a story problem to match a given subtraction calculation using the word "less"	5%	9%

A detailed analysis of the teachers' performance on specific groups of test items is currently being written up. Below, we present some highlights from these data, alongside lessons for future iterations of the course.

4.5.1 Exemplar data and lessons linked to teachers' use of models

Questions 1, 2, 5b, and 6 on the test focused on the use of either the number line or part-part-whole (PPW) model. Here, we use question 6 to highlight some of the issues noted in the detailed analysis of teachers' responses to these questions.

Figure 8. Question 6

Your class is given the question: ___ - 23 = 14. A student tells you they do not know how to figure out what number goes in the gap. How would you help them to solve the problem?

Table 5. Percentage of teachers who gave the correct numerical answer to the question ___ - 23 = 14

	Correctly gave 37 as the answer	Gave incorrect numerical answer	Omitted question or did not give a numerical answer
Pre-test	72%	12%	16%
Post-test	77%	9%	14%

Most teachers were able to give the correct numerical answer to the question in both the pre- and post-test.

Table 6. The different types of explanations offered by teachers for question 6 and the percentage of teachers who gave each explanation

	Just tell them to add	Shows how to calculate 37 - 23	Includes PPW diagram in explanation	Includes number line in explanation	Incorrect explanation	Did not provide an explanation
Pre-test	53%	5%	2%	7%	16%	19%
Post-test	30%	0%	34%	26%	7%	5%

In the pre-test, slightly more than half the teachers indicated that they would just tell their students that they need to add 23 and 14 to get the answer and provided no further justification as to why they would need to do this. Only 25% of the teachers attempted to provide an explanation, but two-thirds of these explanations were completely incorrect, and none of the explanations were full and coherent. In the post-test, 67% of the teachers attempted some explanation, with the majority of these using either a PPW diagram or number line. There was thus a significant shift in the proportion of teachers recognizing the need to provide an explanation as to why to add and attempting the use of some form of model to do so. However, Figures 9 and 10 indicate that the teachers still struggled with the models and with providing a suitable explanation alongside the model. Figure 9 shows that in the pre-test, the one teacher who used the PPW diagram simply produced a PPW diagram with 23, 14, and 37 filled in, without any explanation as to where these numbers had come from. In the post-test, although far more teachers produced a PPW diagram in their answer, only three accompanied it with some form of explanation, and only one provided a full and coherent explanation. In a similar vein, Figure 10 shows that the percentage of teachers using a number line in their answer increased substantially between the pre- and post-tests, but there was still a large proportion of incorrect number lines used and no full and coherent explanations accompanying the correct number lines.

Figure 9. Percentage of teachers providing different answers within "includes PPW diagram in explanation" category

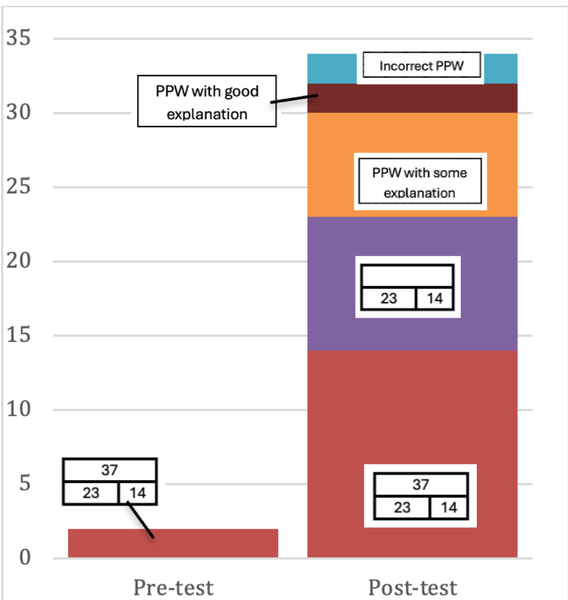
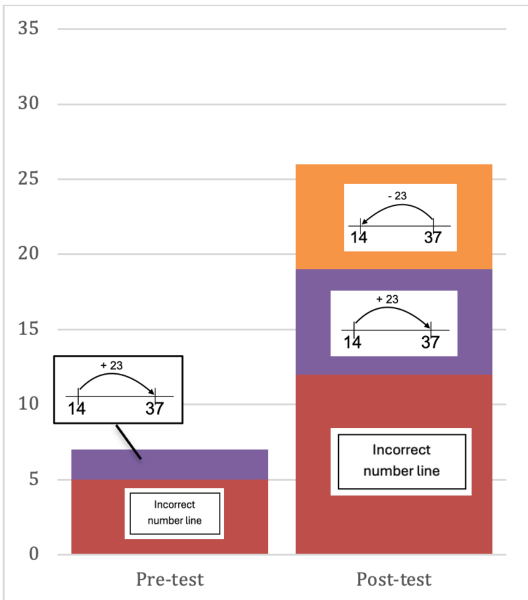


Figure 10. Percentage of teachers providing different answers within "includes number line in explanation" category



This analysis, together with the analysis of the other questions involving work with PPW diagrams and number lines, indicates gaps in teachers' awareness of working with models of situations, as well as more familiarity with the calculations needed to produce answers. The calculations themselves appear to be based on "knowing" what to do rather than on "rationalizing" calculations. The data suggest that the absence of didactically useful explanations will require more attention to creating initial "models of" situations that can then be turned to "models for" calculation, and to demarcating the two.

4.5.2 Exemplar data and lessons linked to language

The way in which teachers presented written explanations in question 6, as well as their construction of word problems in question 7, provided insight into issues linked to language. Here, we provide some data from our analysis of question 7 to exemplify this.

Figure 11. Question 7

Write two story sentences to go with this number sentence: 63 - 7 = __. One story sentence must use the word "less," and the other story sentence must use the word "more."

In Table 4, we saw that only 16% of the teachers could provide an appropriate story involving the word "less" for 63 - 7 = ___ in the pre-test. This rose to 28% in the post-test. Very few teachers (5% in the pre-test and 9% in the post-test) were able to provide an appropriate story problem using the word "more." The difference in performance between the two versions of the question is unsurprising given that subtraction is more easily associated with taking away or having less.

In analyzing the responses (i.e., the kind of sentences the teachers created), it was often difficult to discern whether the difficulty the teacher was having was linked to difficulties with the language demands of English or with connecting a mathematical sentence with a written sentence (irrespective of which written language was being used), or a combination of both.

The three examples below extracted from the teachers' responses illustrate this.

- *John is having 7 balls less than Sipho who is having 63. What is the difference of their balls that they are having?*
- *Sipho has 63 apples he then gives Thando 7. How many less does he have?*
- *Uyanda has 63 sweets. Nothile has 7 less sweets than Uyanda. How many sweets they have altogether?*

None of these stories would lead to the calculation 63 - 7 = ___. All of the sentences have language problems in the way they are constructed. It is not possible to properly understand how the mathematical and language challenges are impacting on each other from the analysis of the written work alone. However, it is clear that language—defined broadly—is an issue that requires greater attention and exploration both in further iterations of the course and in research on it.

4.5.3 Exemplar data and lessons linked to teachers' comfort with working abstractly

Question 4 and question 5a on the test were linked to more formalized or abstract mathematical language. We exemplify some of the issues that teachers' responses raised through the analysis of question 4.

Figure 12. Question 4

The square below is cut out of the hundred square. (A hundred square lists the numbers from 1 to 100 in 10 rows with 10 numbers in each row.)
If I add 21 to the number N, give the letter of the block I will end up in?

N	a	b
c	d	e
f	g	h

Table 7. The different types of answers offered by teachers for question 4 and the percentage of teachers who gave each answer

	Gave correct answer	Gave an incorrect letter as the answer	Gave a number as the answer	Did not answer this question
Pre-test	5%	21%	44%	30%
Post-test	23%	28%	21%	28%

This question required teachers to work with an understanding of the structure of the hundred chart. In the pre-test, only 5% of the teachers could answer the question correctly, and almost half of the teachers provided a number as the answer, suggesting a need to work concretely and a discomfort with generalization and abstraction. There was a substantial increase in the proportion of teachers able to answer the questions correctly between the pre- and post-test test, as well as a decrease in the proportion of teachers who gave a particular number as the answer, indicating the potential for the course to help teachers become more comfortable working abstractly.

Although this kind of question would not necessarily be a focus of the teachers' work with their students in the current program, in Ball et al.'s (2008) terms, this kind of horizon content knowledge is important so that teachers are aware of mathematical trajectories and understand the further mathematics their students will encounter. Students' work with numbers needs to help prepare them for their work with algebra, and thus their teachers need to have an understanding of expressing generality so that they can open opportunities in this regard for their students.

4.6 Data from the Questions Answered by Teachers on WhatsApp

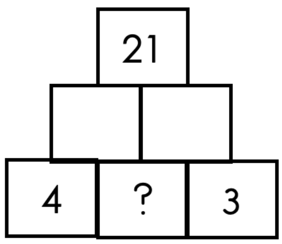
Immediately after the videos for each week was a set of five questions linked to the content of the video. We did not have any mechanisms to prevent "cheating"—that is, teachers helping one another or getting someone else to do the questions—nor did we make any attempt to standardize these quizzes, so we do not consider it useful to look at trends in terms of teachers' marks out of five for the quizzes. However, the data on teachers' responses to each individual questions will be used to decide which of the questions to retain, which to replace, and which to adapt for future iterations of the course.

The limitations of the WhatsApp medium meant that in order to be able to give automatic feedback, we were limited to asking multiple-choice questions or questions that had a simple numerical answer.

The kind of questions we included fell broadly into three of the domains identified as part of MKT by Ball et al. (2008). These were common content knowledge, specialized content knowledge, and pedagogical content knowledge. Below, we provide exemplars of questions from the course for each of the domains and indicate some of the emerging considerations for future iterations of the course that are being flagged by our analysis of teachers' responses to them.

4.6.1 Common content knowledge (CCK) questions

The kind of questions included in this category were often the same questions that the students were asked to solve. Within this domain, there were two categories of questions. The first, exemplified in Figure 13a, were basic calculation questions at the level the students were being asked to solve. Typically, these questions were answered correctly by almost all teachers. However, for the two (out of 28) teachers who gave the answer 2 (= 25 - 23), doing this question and getting the feedback and correction on their incorrect answer might have been very important before they had to guide students on the same task. The other category of questions in this domain were the problem-solving questions that were in the students' worksheet. See, for example, the question in Figure 13b. We felt it important to include these questions because we were concerned that teachers might find them difficult and thus wanted to include them in the WhatsApp course so that teachers could tackle them before their class and get feedback on how to do it if they could not get it right. The fact that 48% of the 25 teachers incorrectly answered the question shown in Figure 13b supports this idea.

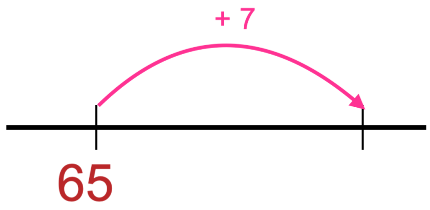
<table>
<tr><th>Figure 13a. CCK basic question</th><th>Figure 13b. CCK problem-solving question</th></tr>
<tr><td>Fill in the missing value
☐ - 25 = 23</td><td>In this pyramid sum, what number should go in the block marked "?"

(Note: In a pyramid sum, the total in each block = the sum of the two blocks beneath it.)</td></tr>
</table>

4.6.2 Specialized content knowledge (SCK) questions

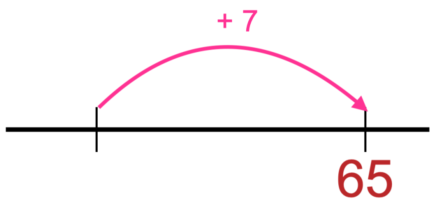
Within the SCK domain, we included questions that checked teachers' understanding of the representations they would be expected to use in the activities. Figures 14a and 14b illustrate the kind of SCK questions we asked teachers in the course. There were two ways in which these questions were useful. First, given that a proportion of the teachers answered these incorrectly, it was a vital check for these teachers before they needed to use these representations in their classes. Half of those who answered the question in Figure 14a chose the incorrect option, and 30% of those who answered the question in Figure 14b chose the incorrect option, so this was a significant portion of the teachers. Second, for the course designers, it allowed us to understand that the representations were not always immediately obvious to the teachers, and so we circled back to explaining their use in future videos and questions in the course.

Figure 14a. SCK question

Which number line represents 65 = ___ + 7?



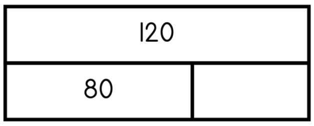
A)



B)

Figure 14b. SCK question

Which story best links to the PPW diagram?



A) I have R80 and get R40. How much money do I have altogether?

B) I have R120 and my mother gives me another R80. How much money do I have now?

C) I have R120 and spend R80. How much money do I have left?

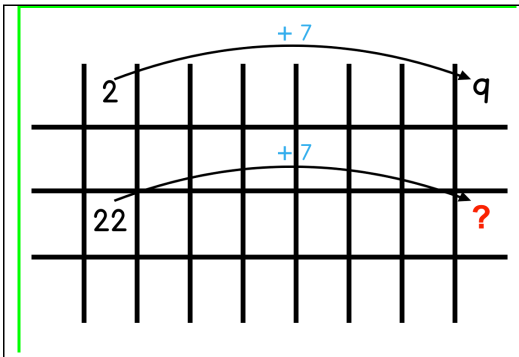
4.6.3 Pedagogical content knowledge (PCK) questions

The PCK questions were the hardest to construct given the limitation of the WhatsApp format. PCK also could potentially encompass a wide variety of ideas and concepts. In our design of the course, we attempted to keep the core pedagogical ideas we were working on with teachers to a small number so that we could regularly circle back to them and reinforce them. We coined the idea of "the 3 Cs" as being the guiding principles in the course.

The first C stands for the Idea that we want to move students on from **c**ounting in ones to **c**alculating efficiently and **c**onfidence in working with numbers. The second C stands for "**C**heck for understanding" and encompasses a few core techniques that we ask teachers to use during class to check if their students are understanding what they are being taught. The third C stands for "**C**onnection," emphasizing the idea that we need to connect new knowledge to prior knowledge and make links between different parts of the lesson—e.g., ensure that if students play a call-and-response game (like the one we termed Fizz Pop) to practice the bonds of ten that they are then reminded to use these bonds in the addition activity that follows.

The PCK questions we included in the course were intended to reinforce these ideas. For example, the questions in Figures 15a and b were intended to emphasize the idea that we want to move students away from counting in ones. In addition, the question in Figure 15b was intended to remind teachers of how they might connect the fluency drill to the activity. Although we had initial concerns that these questions were not sophisticated enough (i.e., it would be obvious which answer we were looking for), it turned out that about half the teachers chose option A or C for both of the questions in Figures 15a and b. Based on this, we believe that in future iterations of the course it might be important to include more of these kinds of questions regularly so that the core ideas are continually reinforced.

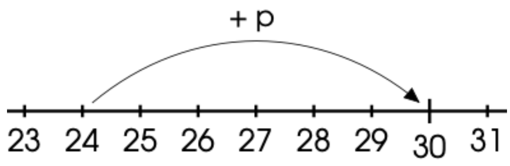
Figure 15a. PCK question



If we ask students to figure out what number goes in place of ?, we would like them to:
A) Count on 7 from 22 i.e. 23, 24, 25, 26, 27, 28, 29.
B) Use the fact that they know 2 + 7 = 9 to say 22 + 7 = 29.
C) Both of these methods are equally good.

Response provided if the teacher answered incorrectly:
Although counting on 7 from 22 will get students to the right answer, at this stage we want them to use the repeating patterns in tens they can see in the hundred chart to do calculations like 22 + 7.

Figure 15b. PCK question



A student doesn’t know how to figure out what p is. You should:
A) Tell them to count up from 24 to 30.
B) Remind them of Fizz Pop adding to get to the next ten.
C) Both of these methods are equally good.

Response provided if the teacher answered incorrectly:
Although counting up from 24 to 30 will get them the correct answer, we want to encourage students to use more efficient methods than counting in ones, so we will remind them that from playing Fizz Pop they know that 4 + 6 = 10, and so 24 + 6 = 30.

4.7 Video Data

Teachers were asked to submit videos aligned to one of the questions from the checkpoints that they administered to the students at the end of each module. The instructions given to teachers for each module, along with the number of videos that were uploaded and some of the key themes emerging from the videos, are shown below.

4.7.1 Video for module 1

Figure 16. Instruction for teachers for video for module 1

Some students probably got the PPW diagrams mixed up for __ - 6 = 9 and 9 - __ = 6. Upload a brief video (1 min max) showing how you would help students understand how to draw the correct PPW diagrams for each of these number sentences.

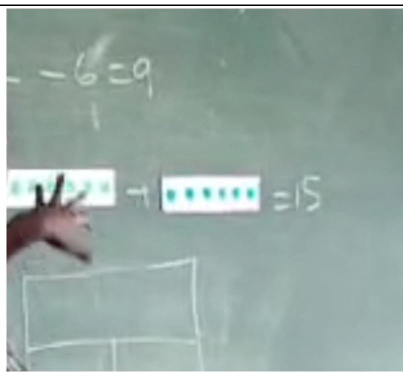
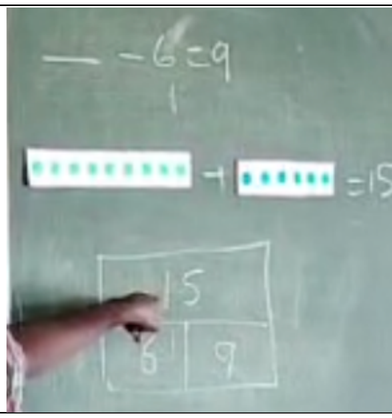
Table 8. Summary of some key responses in video for module 1

Number of teachers who submitted video	Provided an explanation for where to put each of the numbers in the PPW diagram	Used the PPW to explain which operation to use to get the answer	Figured out the missing number and then filled in the PPW diagram	Video uploaded was completely off-task
32	13%	19%	41%	25%

A key focus of module 1 was the relationship between addition and subtraction. In the course materials, the PPW diagram was used extensively to illustrate the relationship and was offered as a tool to help students figure out how to fill in the missing gaps in number sentences like __ - 6 = 9 and 9 - __ = 6.

Outcomes indicated that many teachers continued to struggle with using the PPW to explain which operation to use to get the answer—only 19% were able to do so. A larger proportion (41%) calculated the answer for themselves mentally and then filled in the PPW diagram, thereby treating the PPW diagram as an extra task that needed to be completed rather than as a useful tool for deriving the answer. The video uploaded by teacher T, described in Table 9, illustrates this.

Table 9. Summary of video for module 1 by teacher T

The teacher puts __ - 6 = 9 on the board. She then tells the students that they must start with what they are given and produces a strip with 9 dots and another with 6 dots and then gets students to say that 9 + 6 = 15. Then she places the strips on the board with the + in between them and the answer of 15 shown alongside.	
She then asks the students to put the numbers in the PPW diagram. Pointing to the parts, she asks them what they will put there, and the students reply "6 and 9." She writes these into the PPW diagram. She then asks them, "What is 6 + 9?" and they say "15," which she writes in the PPW diagram underneath. After this, she proceeds to ask the students for the "family facts," which she then lists under the PPW diagram (6 + 9 = 15; 9 + 6 = 15; 15 - 6 = 9; 15 - 9 = 6) .	

The trends from these videos align with the data from the written pre- and post-tests and were further reinforced in the in-depth interviews we conducted with the five selected teachers. These trends reflect an orientation to mathematics that is focused on doing calculations to get the correct answers, eschewing the need to provide a rationale for the calculation.

4.7.2 Video for module 2

Figure 17. Instruction for teachers for video for module 2

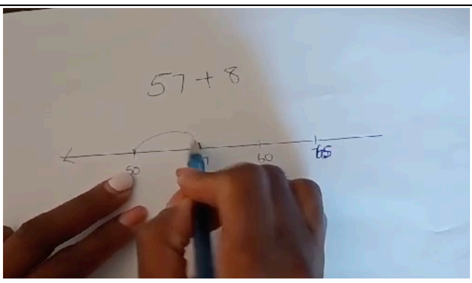
Please upload a brief talking-hand video[2] of how you would show students how to do 57 + 8 using a number line.

Table 10. Summary of some key responses in video for module 2

Number of teachers who submitted video	Knew what to do correctly	Provided a clear and coherent explanation	Explanation was unclear, with extraneous info that detracted from logic or coherence	Video uploaded was completely off-task
30	83%	30%	47%	3%

The majority of teachers were able to use an empty number line to correctly calculate 57 + 8. However, the explanations that these teachers provided often were not logically coherent. For example, some would start by simply saying that the 8 should be split into a 3 and 5, without providing the rationale for choosing that particular split as being about making the jump to the next multiple of ten. Others would add in extra steps without providing a rationale for them, which obscured and possibly confused the important and necessary steps in the calculation. For example, they would say that they would tell students to start by counting up in tens—e.g., 20, 30, 40, 50, 60, 70. This is presumably in order to be clear that 57 lies between 50 and 60, but this is unlikely to be apparent to the students without explanation. The video from teacher W, shown in Table 11, is an example of a video showing a "correct" version of the calculation that nonetheless lacks coherence because it is unclear why one would choose to start at a multiple of ten or why the particular split into 3 and 5 was initially chosen.

Table 11. Summary of video for module 1 by teacher W

<table>
<tr>
<td>"I will start at 50 as a multiple of 10. Then I will break down 8 into 3 and 5, 3 + 5. Then I will go from 50 to 57."</td>
<td>"Then I will take from the 8, then I'll say + 3, then I'll count 3 jumps which will take me to 60 which is the next multiple of ten."</td>
<td>"Then I'm left with a 5, then I'll add 5 to 60. And then I'll say 60 + 5 is equal to 65. Then our answer will be 65."</td>
</tr>
<tr>
<td>
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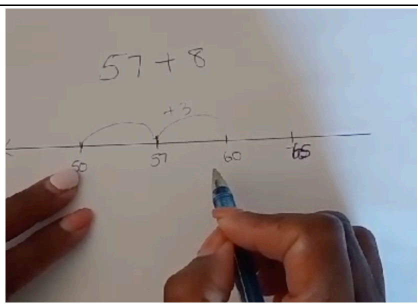
<td>
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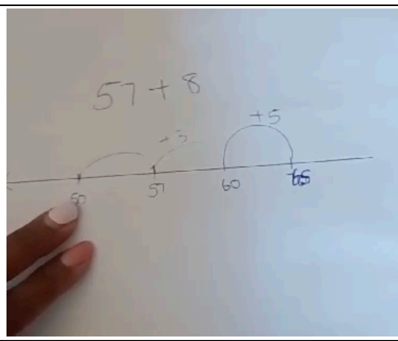
<td>
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</table>

[2] Teachers were shown an example of a "talking hand" video. This is a video clip where the video visuals are focused on what the teacher is writing or drawing (i.e., just their hand and the paper or board on which they are writing), accompanied by a narrative explanation.

A further issue that emerged in the analysis of the videos for module 2 was issues with precision in language. In particular, some teachers talked about jumping to "the nearest multiple of ten" rather than "the next multiple of ten." There was also some potential for confusion in the way in which teachers used "jump"—e.g., "We must make a jump from 57 to 60 which will be three jumps."

4.7.3 Video for module 3

Figure 18. Instruction for teachers for video for module 3

Imagine a student left out the last question: 73 - ___ = 48 and says they didn't know where to start to do it. Please upload a brief "talking hand" video showing the explanation you would give help this student.

Table 12. Summary of some key responses in video for module 3

Number of teachers who submitted video	Got correct answer	Coherently used number line to get missing number	Mostly coherent, but didn't start by making it clear they were aiming at 48	Just did 73 – 48, with no explanation as to why
22	95%	27%	14%	41%

Because module 3 had focused on the number line as a tool, several teachers used the number line in their videos. However, there were two contrasting ways in which the number line was used. In 41% of the videos, the teachers used a number line to illustrate the calculation 73 - 48—i.e., they started at 73 and jumped back 48. In a further 41% of the videos, the teachers aligned their illustrations on the number line with the number sentence 73 - ___ = 48—i.e., they began at 73 and jumped back to 48, taking note of how far they jumped. Approximately two-thirds of these explanations were clear and coherent, motivating and linking the number sentence to the number line well. The explanations in the other third matched the number sentence but launched in on the jumping back from 73 before explaining clearly that the purpose was to see how much needed to be subtracted from 73 to get back to 48.

Again, we see that teachers were able to answer the question correctly. However, the propensity for a large proportion of them to simply do the calculation 73 - 48 without any justification as to why they would do that calculation aligns again with an orientation toward mathematics as being about knowing what to do rather than being able to explain why it works that way.

4.7.4 Summary of the lessons drawn from the video data and implications for future iterations of the course

The key lessons we learned from the brief videos that teachers provided at the end of each module can be summarized as follows:

- Teachers focus on telling the steps of what to do, with little focus on the rationale for those steps.

- Models (e.g., the part-part-whole diagram or number line) are seen as tasks to complete rather than as tools for understanding.
- Teachers have difficulties using age-appropriate language and providing coherent explanations.

These were valuable insights for us and provide indications of the kind of skills we might want to focus on more in the in-person training sessions and issues we might need to tackle more directly and clearly in the videos we create for teachers.

The one quandary that requires further thought is the fact that watching and analyzing these videos is time-consuming. Although we tried to give some general feedback to the teachers on issues that we picked up in their videos collectively, we often could not do that in a timely manner and certainly were not providing individual feedback to the teachers, which we imagine might have been more valuable. This quandary would become even more acute if the course were run with a larger group of teachers. As it stands currently, these data are useful for the design of further tasks for teachers but not for individualized feedback.

4.8 In-Depth Pre- and Post-Interviews with Five Teachers

Five teachers (one from each of the regions) were interviewed for about an hour at the start of the course and then again toward the end of the course. The pre-interviews included a discussion of the teachers' experience teaching mathematics and their attitudes toward mathematics, as well as some task-based discussion. The post-interviews covered the teachers' experience with the after-school program and with the WhatsApp course, as well as task-based discussion. Both the discussion of teachers' experiences and the task-based parts of the interviews align with the broader-level teacher data discussed above. These interviews have given us greater insight into some of the issues with models and explanations that we saw from both the written pre- and post-tests, as well as from the end-of-module videos the teachers uploaded. For some of the teachers, the initial analysis seems to indicate some growth in their understanding over time. However, for others, their difficulty in working with models as a tool for understanding persisted, and it appears that a strong orientation toward mathematics as telling students about what the steps are in a calculation (without rationale) created difficulty in assimilating some of the teaching ideas promoted in the course.

Our in-depth analysis of these interviews—which, as noted earlier, is currently being drafted—complements the key findings discussed in this report and provides further detail on and texture to the issues raised above. We will provide a link to the journal article in which this analysis is being written up once it is published.

5. Discussion and Next Steps

In this initial iteration of the WhatsApp MKT course over the first half of 2024, we saw an improvement in teacher MKT between pre- and post-tests, as well as positive responses from teachers about the usefulness of the course. Although we experienced some challenges with the technology, we still consider WhatsApp to be an accessible and affordable platform for teachers to access content, and both we and turn.io (which runs the underlying platform that we use to host the course) have been improving the software to allow for a better user experience both from the point of view of teachers who need to access the course and from the perspective of managers and instructors who need to monitor teachers' engagement with the course. The research process accompanying the rollout of the course has been very useful in pointing to key areas for improvement—some of which were implemented immediately and others which are under development for the second iteration of the course in 2025. In what follows, we summarize the important lessons that we have learned and indicate their implications for the course going forward:

- *Less is more*: Multiple apps and too many tasks to do simultaneously is overwhelming for course participants; therefore, it is important to keep the design simple and the amount to be done to only what is necessary. In addition, it took a while for teachers to understand the purpose and elements of the course, so repeating the same structure each week was useful.
- *In-person onboarding is essential*: Despite teachers' familiarity with the WhatsApp environment, a number of them struggled at first to access and understand the course. We will use a portion of the initial in-person training to help all teachers register for the course and run through the first lesson so that we can iron out any difficulties before they need to work on it remotely.
- *Teacher take-up needs to be encouraged*: Despite the fact that the weekly content in the course was focused on helping teachers prepare for what they would be teaching in the coming week, teacher take-up was not as a high as we wanted it to be. For future iterations, we are considering including gamification elements to incentivize teachers, as well as enabling easy access to lesson completion data for managers and coaches.
- *Many teachers' experience with mathematics and models is different from what we are asking them to embrace*: The purpose of the key models used in the course was not necessarily apparent to teachers. The teachers are more familiar with providing the steps to do calculations than with working with models of situations to provide rationales for the calculations that need to be applied. The way we introduce these ideas to teachers and work with them throughout the course needs to be done with the awareness that for many of them this will require a significant shift in their thinking.
- *Coherent explanations are hard to construct*: For many teachers, constructing coherent explanations of mathematical ideas, particularly ones linked to models that might not be completely familiar to them, was challenging. We need to find ways to help teachers do this and mechanisms to allow them to practice and get feedback on their explanations.
- *We need to understand more about the role of language*: Language emerged as an issue in the course in a number of instances, but the data did not provide enough insight into the exact nature of the issue, which by its nature is likely to be a complex interplay of factors linked to mathematical understanding, mathematical language, and the medium of instruction and teaching. However, it is clear that in the next version of the course this issue needs to be a strong focus both of the research and in our interactions with teachers.

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