Materials Guidance for Numeracy Programs

PART 1: Selecting and Using Materials – The Basics

INTRODUCTION

As described in the Science of Teaching structured pedagogy how-to series, teaching and learning materials play an important role in supporting children's learning. At the same time, materials can represent a significant resource investment, so making decisions about what materials to use and how to ensure that they are used appropriately is an important part of designing a numeracy program.

This guidance is intended to help decision-makers such as program implementers and ministry officials understand and make decisions about adopting and incorporating materials for early grade math programs. It also provides information for curriculum and materials developers and teacher trainers, who play a key role in ensuring that materials are used appropriately to support children's learning.

In keeping with this dual purpose, the guidance is presented in two parts. This first part provides general information on the selection and use of materials, including print materials, concrete and pictorial math models, and education technology. The second part provides more in-depth information about manipulatives and pictorial models that are commonly used in math programs.

Part 1 begins by presenting key considerations for using written materials—including teacher guides and student books—in the math classroom. It then focuses attention on manipulatives and other models that help children understand math concepts, explaining the importance of these manipulatives and models and providing recommendations on the essential ones to include in a math classroom toolkit. Considerations around acquiring or developing materials, as well as preparing teachers to use them appropriately, are discussed. Finally, it ends with a table listing common manipulatives and models and the math domains or topics for which they can be used.

Definitions

Abstract
A mathematical concept represented through symbols (e.g., the addition sentence 4 + 2 = 6)

Competency
A statement that describes the desired knowledge or skill for a student to gain

Concrete
A mathematical concept represented using physical objects (e.g., counters)

Domain
An area of study in a curriculum (e.g., number sense, operations, measurement)

Manipulatives
Physical materials that can be moved and touched, such as counters

Pictorial
A mathematical concept represented through a picture, drawing, or figure

Representation
A mathematical concept or idea shown through symbols (abstract), drawings (pictorial), or objects (concrete)

Design and Adoption of Teacher and Student Books

Written materials, such as teacher guides and student books, may be developed or adopted by programs to support math teaching and learning.

When making decisions about books, the purpose that these books serve for instruction should be kept in mind. Student books and teacher guides should provide opportunities and support for:

- Students to attempt to make sense of a new problem or to solve a new problem—often at the start of a lesson—individually or in small groups
- The teacher and students to jointly discuss problem-solving strategies
- Students to practice solving additional problems independently, whether individually or in small groups
- Students to have hands-on practice with concrete and pictorial materials when new concepts are introduced

Written materials should also provide opportunities for routine checks for understanding and incorporate opportunities for remedial practice (for struggling students) and/or enhancement (for more advanced students).

There is no one lesson structure or book format that has been shown to be most effective in all contexts and programs—these have in fact varied across successful math programs. However, there are some elements that are common across such programs and are likely to support the purposes mentioned above:

- The student book design should provide students with multiple opportunities to access the same content through different activities, using different concrete and pictorial representations—such as showing addition using both pictures of objects and a number line.
- References to concrete materials in the student book and teacher guide should consistently reflect an available and appropriate set of materials that teachers are trained to use (the following pages provide guidance on material selection and training).
- Teacher guides should provide guidance for teachers on how to facilitate opportunities for students to practice, using concrete and pictorial materials themselves, independently and in small groups.
- The teacher guide should provide guidance or support for teachers on key instructional strategies for mathematics, including the following: linking formal and informal mathematics; discussing mathematics; using appropriate models and representations; and using knowledge of students and learning progressions to target instruction (for further information on these strategies, see the How-to Guide on Designing Effective Numeracy Programs).
- The design of the student book should be age and developmentally appropriate and reduce reliance on reading and language skills. In student books, text should be minimized in the early years and aligned to the expected literacy level of students. Images that are not related to the mathematical content should be minimized.
- The teacher guide design should be consistent from lesson to lesson, should be aligned to the student book, and should use diagrams or photos to support teachers in using accurate concrete and pictorial representations. It should also be consistent in style and approach with other guides that teachers use.

Workbooks or no workbooks?

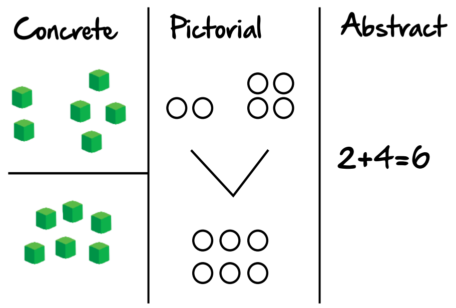
This question is often raised. Student workbooks, books that students can write in, can make it easier for young students to practice math activities without having to copy from another book or board (which can be a time burden and result in mistakes). They can also help guide key early math activities, such as tracing numbers or coloring shapes. However, they need to be replaced each year and so can be more expensive. Many programs provide workbooks for grade 1 and then transition to using only textbooks in grade 2 or 3.

For more about **designing teaching and learning materials**, refer to the Science of Teaching how-to guides on structured pedagogy, including Guide 4, *Teaching and Learning Materials Development:* https://scienceofteaching.site/structured-pedagogy/#how-to

The Importance of Representations: Concrete, Pictorial, and Abstract

Using different representations, and especially beginning with concrete representations, helps children understand mathematical concepts.[1] Research has shown that using concrete materials in the math classroom can have positive outcomes on learning.[2] Using concrete and pictorial representations to introduce and build a concept, and then moving to the abstract, helps ensure that children can understand and apply the abstract concepts.

FIGURE 1. Learning about addition using manipulatives, pictures, and then symbols



Manipulatives are concrete materials that can be moved and touched. They include counters, fingers, place value sticks, and geometric shapes (cutouts). Using manipulatives helps children make sense of math concepts that would otherwise be abstract to them. In the early primary grades, children should usually work first with manipulatives when learning a new topic. This prepares them to work next with pictorial materials and finally abstract symbols. In the example shown in Figure 1, the children first work with counters to see how they can put two groups of blocks together and figure out how many they have altogether.

Manipulatives are most likely to result in better learning outcomes if all students have the opportunity to use manipulatives themselves, rather than simply observing the teacher or other students use them.

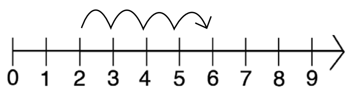
FIGURE 2.

Using number line for addition

2+4 =

What is 4 more than 2?

Start at 2 and count on 4:

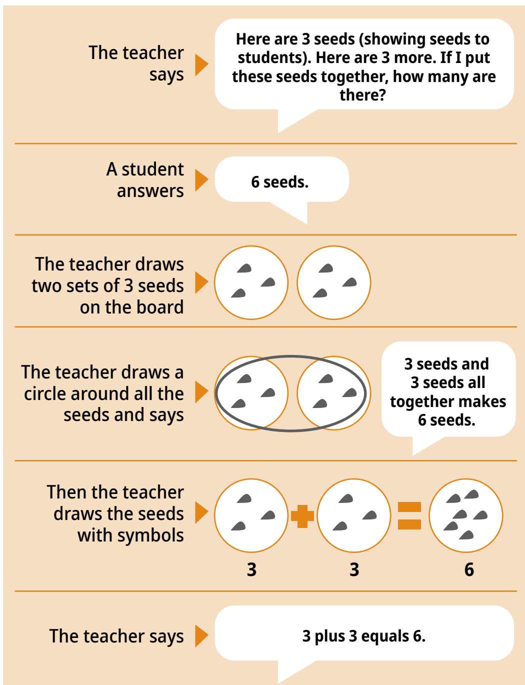


Pictorial materials model math concepts in visual ways, such as through a drawing, diagram, or chart. They also include representations such as the number line, 100 chart, place value chart, and fraction shapes. These materials can help children bridge from the concrete to the abstract and, in some cases, can provide an alternative or additional way of thinking about a concept, which may help students understand better. For example, in Figure 2, a number line is used to show the same addition problem (2+4=6). Using this model in addition to the one presented in Figure 1 can deepen children's understanding of addition and also give them another strategy to use.

Abstract representations show mathematical concepts using numbers and symbols. The use of symbols in mathematics is what lets us apply mathematical concepts to new situations and new problems—so we want children to be able to use them and understand their meaning. In Figure 1, the concept of addition is first shown using concrete and pictorial representation and then using abstract symbols. The goal is for children to understand and feel just as comfortable with the abstract representations as they do with concrete materials and pictures—though they may sometimes want go back to using objects and pictures to aid in their thinking.

FIGURE 3.

Explicitly linking concrete, pictorial, and abstract representations



It is important that the teacher clearly demonstrate the connection between concrete, pictorial, and abstract representations. Figure 3

1 M. K. Stein and J. W. Bovalino, "Manipulatives: One Piece of the Puzzle," *Mathematics Teaching in Middle School* 6, no. 6 (2001): 356–360.
2 P. Swan and L. Marshall, "Revisiting Mathematics Manipulative Materials," *Australian Primary Mathematics Classroom* 15, no. 2 (2010): 13–19.

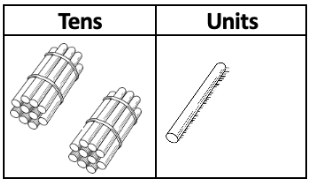
shows an example of how a teacher might do this (based on observations of the TAFITA program in Madagsacar).[3] Teachers should also ensure that students move on to using abstract representations at an appropriate point. For example, using counters or their fingers is appropriate for the early grades, but children should be able to do simple addition using only abstract symbols by the end of grade 3 or earlier. Teachers can use concrete and/or pictorial representations in later lessons or grades for students who cannot yet add using abstract symbols. Once students are able to add consistently using concrete and pictorial representations, however, the teacher should encourage them to transition to solving problems without them. The teacher should model different strategies they might use to solve a problem abstractly, and ask students to try these strategies. As children progress through different topics in math, the concrete-pictorial-abstract process may be repeated. For example, students may be at the abstract stage with whole number addition, but will return to the concrete stage when being introduced to fraction addition.

Selecting Appropriate Materials

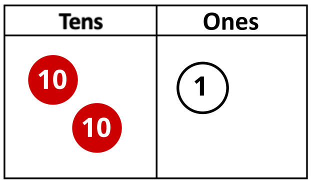
Materials should be selected based on their conceptual and contextual appropriateness. A **conceptually appropriate** material is one that can be used to show the concept accurately and is effective in teaching the math topic at the level required. For example, Figure 4 shows appropriate and inappropriate materials for teaching about place value. The sticks and bundles are more appropriate for teaching this concept because they clearly show, and children can easily understand, that ten sticks can be bundled together to make a 10. While teachers could tell young children that a white disk means 1 and a red disk means 10, this does not show the concept clearly and is likely to be confusing to them. Similarly, bundles can be physically taken apart and used to demonstrate the concepts of regrouping (sometimes called "carrying" and "borrowing"). In higher grades, once children have a good understanding of base 10, there are activities for which these disks could be appropriate, but they are not recommended for early grades.

FIGURE 4.

Appropriate for teaching place value in early grades



NOT appropriate for teaching place value in early grades



A **contextually appropriate** material is one that is appropriate for the local context in terms of its accessibility in the local environment and its applicability to the local curriculum. Teachers' existing familiarity with a material may also make it more contextually appropriate. The local curriculum should be considered when selecting a set of materials to ensure that the set is sufficient to cover the content. There may also be specific materials recommended by the local curriculum that teachers are already familiar with. Other realities of the local context should also be considered when choosing or procuring manipulatives, such as the numeral-writing system, currency, and common tools used for measurement locally. Additionally, beans are sometimes used to teach counting, and oranges are sometimes used to teach fractions—however, these items are not appropriate in a context where children come to school hungry. Student safety should also be considered. For example, while matches may seem useful to use as counters and to make bundles of ten, the possibility that children will put them in their mouths or even light them should be considered.

FIGURE 5. Using bottle caps as counters

It is important to select materials in a strategic way. A set of materials for a math class should include a relatively small selection of materials that are affordable or easy for teachers to create. For example, Figure 5 shows students in the RAMP program in Jordan using bottle caps as counters—a material easy to find in the local context.[4] These materials should also be easy to store, carry, and use throughout the year. Keeping the materials to a manageable number also reduces the number of new instructional

3 W. Ralaingita, M. Stern, J. DeStefano, and Y. Sitabkhan, "Numeracy at Scale Findings Brief: TAFITA Program in Madagascar" (June 2023), https://learningatscale.net/numeracy-findings/.
4 J. Stern, W. Ralaingita, and Y. Sitabkhan, "Numeracy at Scale Findings Brief: RAMP Program in Jordan" (June 2023), https://learningatscale.net/numeracy-findings/.

techniques that teachers are required to know.[5] A single material can be used to teach multiple topics. For example, counters can be used to count, add, subtract, multiply, and divide numbers. At the same time, a single math topic can be taught with different types of materials, which can help provide children with alternative ways of thinking about a concept—thus enabling more children to understand. When teaching counting, for example, it is helpful to use both a concrete tool (e.g., counters) and a pictorial tool (e.g., a number line).

ESSENTIAL MATERIALS

Programs may have budgetary and planning limitations, but they should ensure that teachers have a set of essential materials that is sufficient to deliver the curriculum, including core concrete and pictorial materials.

Concrete materials are critical when learning how to count and identify the place value of numbers. It is recommended that all programs use counters and a concrete material for place value, such as sticks.

Pictorial materials can build and solidify the concepts students develop using concrete materials, as well provide an alternative way of showing the concept to deepen children's understanding. Pictorial materials can be included in the student book, displayed on a poster, or drawn on the board or on paper (possibly laminated for reuse).

The minimum recommended set of essential concrete and pictorial materials is as follows:

Concepts	Concrete materials	Pictorial materials
Counting/Number recognition and simple operations	1 concrete material (such as counters)	2-3 materials (e.g., pictures of objects, 100 chart, number line)
Place value and operations	1 concrete material (e.g., sticks and bundles or base 10 blocks)	2-3 materials (e.g., place value chart, ten frame, 100 chart)
Fractions		Fraction strips, fraction shapes*
Measurement	Ruler**	
Geometry		Geometric shapes***

*Fraction strips or fraction shapes can also be cut out to be used as manipulatives.
**Rulers can be store bought or can be printed in the back of a student book.
***Geometric shapes can also be cut out to be used as manipulatives.

The end of this guide includes a table that shows a range of concrete and pictorial materials that may be considered, which are also discussed in detail in Part 2.

Providing or Obtaining Materials

Materials may be provided by a program or obtained or created by teachers or students. There is some evidence that providing teachers with the materials they need can help ensure that they are appropriate, ample in number, and likely to be utilized.[6] They can be provided in a box or bag, which helps teachers store the materials safely and transport them to class. In some cases, teachers or students may be asked to find or create their own materials and to be responsible for storing

5 B. Piper, Y. Sitabkhan, and E. Nderu, "Mathematics from the Beginning: Evaluating the Tayari Pre-Primary Program's Impact on Early Mathematic Skills," *Global Education Review* 5, no. 3 (2018): 57–81.
6 Y. Sitabkhan, J. Davis, D. Earnest, et al., "Instructional Strategies for Mathematics in the Early Grades," Mathematics Working Group Working Paper (Washington, DC: USAID, 2019), https://shared.rti.org/content/instructional-strategies-mathematics-early-grades.

and transporting them to class. Implementers should consider the extent to which teachers and students are willing and able to procure, store, and transport the materials when making decisions and providing guidance on materials.

When teachers or students are expected to obtain materials themselves, they should be easy to find or make using materials from the local community. This means that they should not have a cost and should take a minimal amount of time to create. Programs can identify appropriate materials by examining the resources that are available in communities. Implementers can also make a set of the materials themselves to get a sense of how long this process might take teachers and students.

Whether they're provided by the program or obtained by teachers and students, materials should be sturdy enough to be used repeatedly throughout the year. Paper materials such as number cards and fraction shapes can be printed or drawn on a thick material such as cardboard or taped/glued onto the thick material.

Teacher Training on Materials

The accurate use of materials has been shown to improve skills ranging from counting to problem solving.[7] Conversely, if used inappropriately, materials may "result in frustration or confusion, and ultimately disrupt student motivation and overall learning opportunities."[8] Providing materials to classrooms and teachers without instruction or guidance has minimal impact on learning outcomes.[9] Teachers should receive training on the materials that they are expected to use, including both manipulatives and written materials such as teacher guides.

If teachers are expected to create their own materials, then teacher training should include guidance on doing so. Training sessions can be opportunities to have teachers create materials to use in their classroom with provided resources, and practice with the materials they have created. Sufficient practice with a material will help teachers gain fluidity in using it appropriately. It is also important to offer support throughout the academic year—as teachers progress to more advanced topics, they need to try out new materials or try using familiar materials in new ways.

In-service training and ongoing support can be planned to align to the curriculum taught at different points during the academic year. This can ensure that sufficient time and resources are dedicated to each material that teachers are expected to use. For example, if numbers and operations are taught during the first term, then the initial teacher training may focus on how to teach these topics using materials such as counters and the number line. If measurement is taught in a later term, then mid-year training or ongoing support may be planned to address how to teach measurement using materials such as a ruler or model clock.

To use materials effectively in lesson activities, teachers should be able to:

- Select appropriate materials for the math topic and activity
- Identify how many materials are needed
- Prepare materials for themselves and students
- Manage distribution and collection of materials
- Model mathematically correct and appropriate use of the materials
- Clearly show and explain math concepts to students using the materials

7 Clements (1999), cited in N. Evans, D. Srikantaiah, A. Pallangyo, et al., "Towards the Design and Implementation of Comprehensive Primary Grade Literacy and Numeracy Programs," Global Reading Network Working Paper (Washington, DC: USAID, 2019).
8 R. S. Liggett, "The Impact of Use of Manipulatives on the Math Scores of Grade 2 Students," *Brock Education Journal* 26, no. 2 (2017): 88.
9 Evans et al., "Towards the Design and Implementation," 2019.

- Guide and monitor students' use of the materials
- Plan for the storage and reuse of materials[10]

Those responsible for monitoring and supporting teachers, such as coaches or principals, should also be trained in and knowledgeable of best practices in using materials, so that they are able to observe and provide feedback to teachers.

> For more about **planning teacher training on materials**, see the Science of Teaching how-to guides on structured pedagogy, including Guide 5, *Teacher Professional Development: Teacher Training* and Guide 6, *Teacher Professional Development: Ongoing Teacher Support:* https://scienceofteaching.site/structured-pedagogy/#how-to
>
> For more about **numeracy programs and the role of materials and teacher training**, see the Science of Teaching how-to guide, *Designing Effective Numeracy Programs in Low- and Middle-Income Countries*: https://scienceofteaching.site/numeracy-program-design-and-implementation/#how-to

Education Technology and Online Resources

Programs may choose to incorporate education technology (EdTech) to support teaching and learning. EdTech can be integrated in math education in several ways, including as instructional materials and resources, as a tool to support collaboration or communication, to support monitoring and assessment, or for adaptive math practice.[11] Evidence to date suggests a particularly positive effect of technology in supporting the practice of discrete skills, problem solving (e.g., through game-based or personalized interactive software), and conceptual development in certain topics (e.g., through virtual manipulatives, such as dynamic geometry software) in mathematics.[12] While much of the research on technology and mathematics stems from higher-income countries, positive outcomes of EdTech for mathematics learning, especially through personalized learning software, have also been found in low- and middle-income countries.[13]

When choosing EdTech resources, many of the same considerations apply regarding contextual and conceptual appropriateness. As with the concrete and pictorial materials described elsewhere in this guide, EdTech resources should be user-friendly and support teaching and learning of the local curriculum. There are additional factors to consider when introducing EdTech, however, including the availability of devices, electricity, or internet, depending on the specific EdTech tool or application. Besides infrastructure, user support is essential for the scale-up and sustainability of the program. Teacher training will also play a critical role in the effective use of EdTech for early mathematics. This includes not just training in operating and navigating the EdTech resource, but also training on its selection and use to ensure pedagogically valuable use.[14] The use of EdTech can also exacerbate existing educational inequalities if it is part of the program but only useable by some beneficiaries, such as those in urban or private schools. Yet, studies in the US find particularly promising effects of technology in mathematics for lower-performing students.[15] It can therefore be helpful to conduct a review of existing EdTech strategies in the local context and conduct a pilot before implementing EdTech as part of a program.

Research also suggests that implementation plays an important role in relation to impact, and thus the benefit of EdTech integration will vary and depend on the context. Any one product may yield

10 S. Lutfeali, Y. Sitabkhan, W. Ralaingita, & B. Piper, *Designing Effective Numeracy Programs in Low- and Middle-Income Countries* (RTI International, 2021), https://scienceofteaching.site/wp-content/uploads/2023/08/PROJ_12_SoT-Practical-Guide-for-Numeracy_15AUG23.pdf.
11 H. Ran, N. J. Kim, & W. G. Secada, "A Meta-Analysis on the Effects of Technology's Functions and Roles on Students' Mathematics Achievement in K-12 Classrooms," *Journal of Computer Assisted Learning* 38, no. 1 (2021): 258–284.
12 Ibid.
13 L. Major, G. A. Francis, & M. Tsapali, "The Effectiveness of Technology-Supported Personalised Learning in Low- and Middle-Income Countries: A Meta-Analysis," *British Journal of Educational Technology* 52, no. 5 (2021): 1935–1964.
14 E. Arias Ortiz, J. Cristia, & S. Cueto (eds.), *Learning Mathematics in the 21st Century: Adding Technology to the Equation* (Inter-American Development Bank, 2020).
15 Ran et al., "A Meta-Analysis," 2021.

different outcomes in different implementations and contexts, and research that could examine implementation effects is still nascent.

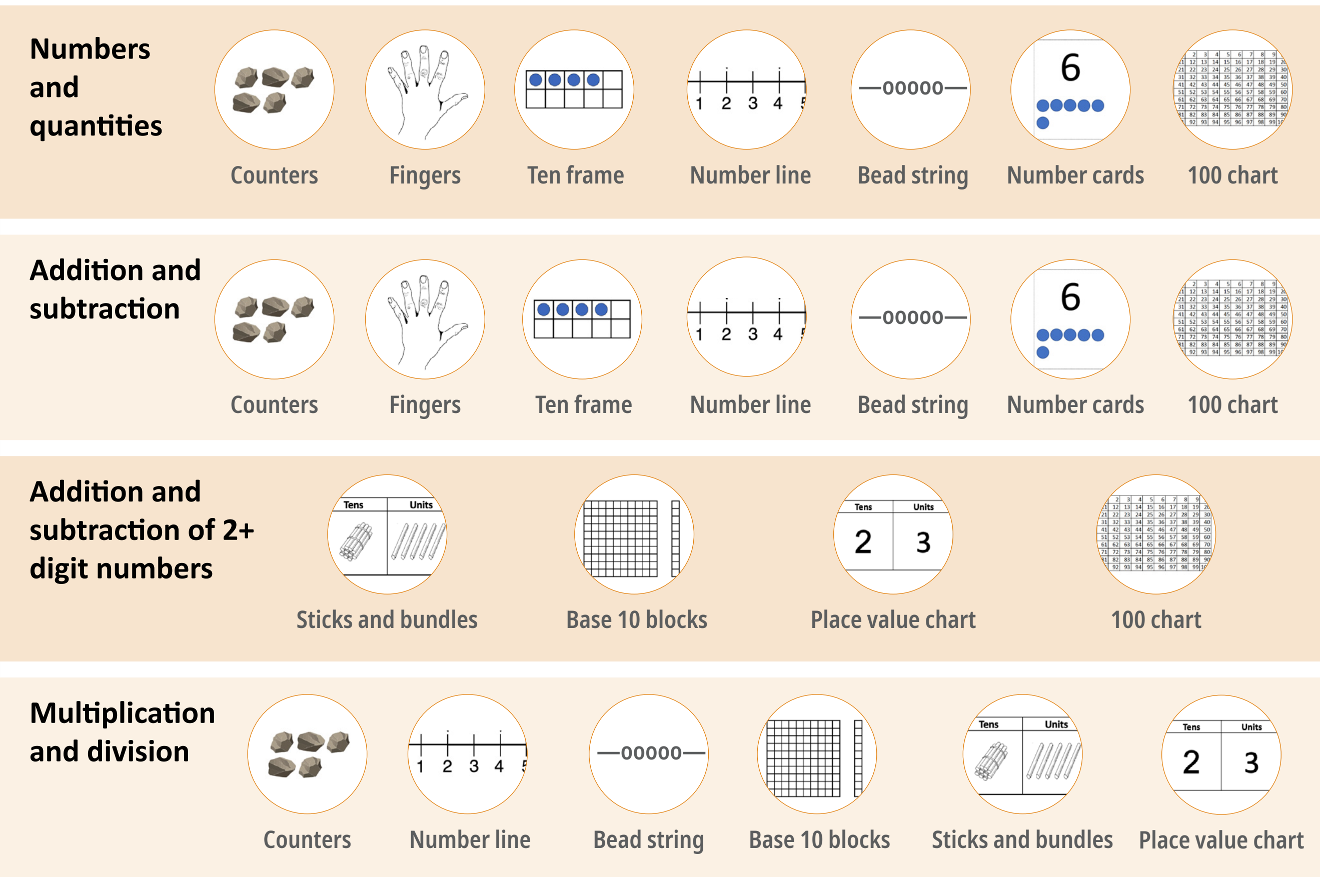
The following resources provide more information about technology in education, including in numeracy:

- EdTech Hub – This website compiles and shares evidence on EdTech interventions, as well as resources and tools.
- EdTech Tulna – This website provides a set of standards for evaluating EdTech products and undertakes and shares evaluations of existing products.
- Learning Mathematics in the 21st Century: Adding Technology to the Equation – While this report focuses on the Latin America and Caribbean region, it includes information useful for sub-Saharan Africa and elsewhere. The report includes a clear and useful discussion of how technology can support math learning, current realities, and evidence from implementation examples in the region.
- Using Education Technology to Improve K-12 Student Learning in East Asia Pacific: Promises and Limitations.

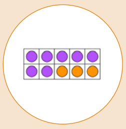
Common Manipulatives and Models

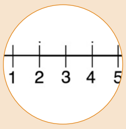
Figure 6 provides an overview of common concrete and pictorial materials, presented according to the mathematical topics they can be used to support. There may be alternative materials that teachers are already familiar with, and the considerations noted in this document can help guide decisions around their use. More detailed information on each of these materials can be found in Part 2.

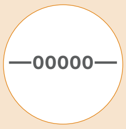
FIGURE 6. Common manipulatives and models

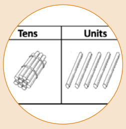


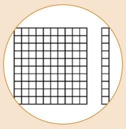
Place value


Ten frame


Number line

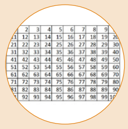

Bead string


Sticks & bundles

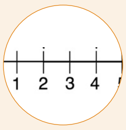

Base 10 blocks

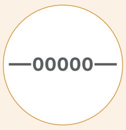

Place value chart

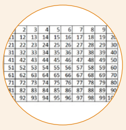

Number cards


100 chart

Patterns


Number line

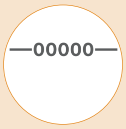

Bead string

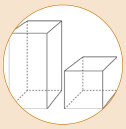

100 chart


Counters


Number cards

Measurement


Bead string


Geometric shapes


Ruler

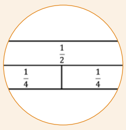

Model clock (time)

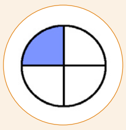

Calendar (time)


Model money (money)

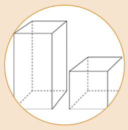

Everyday objects

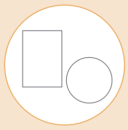
Fractions


Fraction strips


Fraction shapes

Geometry


3D shapes


2D shapes

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