Knowledge Progressions and Connections: Numeracy

INTRODUCTION

High-impact teaching strategies (HITS), also referred to as high-leverage practices, are core pedagogical practices that help students understand content while also supporting their social-emotional development.[1] These teaching practices can demonstrably impact student learning outcomes in both literacy and numeracy.[2]

This mini-guide, one in a series, sheds light on how teachers can apply the high-impact teaching strategy of knowledge progressions and connections in their numeracy lessons.

HOW TO USE THIS GUIDE

This guide is meant for those who support teacher professional development at the school level. Depending on the local context and approach to professional development, this role can be played by coaches, community of practice leaders, teacher-facilitators, lead teachers, trainers, and Ministry of Education staff working with teachers. As part of a wider professional development approach, this guide can be a tool to target teachers' use of proven strategies to improve student learning in literacy and numeracy.

Teacher professional development has various modalities, ranging from pre-service courses to one-on-one coaching and mentoring to teacher-led communities of practice. This guide is intended to support professional discussions across a variety of contexts: in-school coaching, communities of practice, and pre- or in-service training. Teachers and coaches can use the guide as part of an individualized professional development plan; teacher-facilitators can use it to drive discussion on high-impact strategies as part of a community of practice; and pre-service designers can incorporate it into their curriculum for teacher training. Finally, this mini-guide reflects the incremental progression that teachers follow in their professional growth.

This mini-guide for Knowledge Progression and Connections in Numeracy, along with the others in the series, reflects the strategy domains and the HITS presented in the following table.

Finally, when adapting the content and use of this mini-guide to the local educational context, it can be integrated into a structured pedagogy program already in place. The mini-guide can be linked directly to the curriculum and teaching and learning materials, and supported through the existing professional development model.

1 Ball & Forzani (2010).
2 Ambrose et al. (2010); Danielson (2022); Rosenshine (2012).

HIGH-IMPACT TEACHING STRATEGIES (HITS)				
Strategy domains		**Tier 1: Basic strategies**	**Tier 2: Developing strategies** *(built upon Tier 1 strategies)*	**Tier 3: Advanced strategies** *(built upon Tier 1 and 2 strategies)*
Learning Environment		Teacher establishes clear **rules and routines** to support learning and create a positive learning environment.	Teacher uses and supports **positive interactions** in the classroom (teacher-student and student-student).	Teacher organizes students to work in **pairs and small groups** to enhance collaboration, to build teamwork, and to promote a sense of belonging.
Student Engagement in Learning		Teacher gives all students, including pairs/small groups, **regular time for the practice of new skills**.*	Teacher uses **questioning** to build and deepen student understanding of new content.	Teacher plans for the strategic use of **partner and small-group work** for collaborative learning activities.**
Knowledge Progression and Connections		Teacher plans with and states the **lesson objective** and links new content to **students' background** (prior knowledge).	Teacher provides a **daily review** and links content to **previous learning**.	Teacher purposefully **sequences** lesson objectives and **adjusts the teaching sequence** as needed.
Assessment-Informed Instruction		Teacher routinely **monitors learning** by checking for understanding during instruction and giving actionable feedback to students.	Teacher **modifies content and instructional strategies** based on evidence of learning collected through formative assessment.	Teacher provides **differentiated instruction and remediation** to address learning gaps.
Instructional Approach	**Numeracy**	Teacher **provides explicit models and explanations** of math concepts and skills, followed by **student practice** of modeled skills.	Teacher uses questioning and other **interactive approaches to build student understanding** when **modeling and explaining** math concepts and skills. *Modeling/explanation is followed by student practice.*	Teacher provides **opportunities for students to explore** concepts and then **draws on their ideas when modeling and explaining** concepts and their application.
	Literacy	Teacher provides **explicit models and explanations** of new skills and concepts, followed by **student practice** of modeled skills.	Teacher **adds activities** to **construct meaning (or build knowledge)** together with students.	Teacher gives students **opportunities to apply skills in meaningful ways**.

Notes:
*Tier 1: Teacher groups students to engage all children in the learning activity, especially when materials are being shared.

**Tier 3: Teacher purposefully groups students to engage all children through homogenous or heterogenous grouping. Homogenous grouping can be used with students working at a similar, medium level to learn at a higher level together. Heterogeneous grouping is used to provide peer support to students who may be struggling with new content and skills.

Recognizing that professional development is most effective when it is focused and incremental, this guide proposes a tiered approach to mastering a collection of strategies, as shown in the table. Professional development activities that support teachers, especially those working in low- and middle-income countries, in mastering the specific strategies listed under Tier 1 are likely to result in improved instruction and learning outcomes. As teachers master the basic strategies of Tier 1 and move toward the more "advanced" Tiers 2 and 3 in each category, they will be empowered with a full set of strategies to reach and support more students. Over time (as measured in years, as opposed to weeks or months), a professional development program that follows a tiered approach can help teachers move toward more depth of instructional mastery (higher tiers) and significantly improve the quality of their teaching, which will ultimately help more students develop a deeper understanding and mastery of foundational literacy and numeracy skills and concepts.

It is important to note that not all teachers will start at Tier 1. Depending on the teacher's level of experience and familiarity with implementing the strategies, she may start at Tier 2 or even Tier 3. Furthermore, a teacher starting at Tier 2 within the Learning Environment domain may start at Tier 1 in the Knowledge Progression domain; this type of variation and personalization is a key component of this guide and reflects each teacher's individual journey within classroom practice. Each guide includes a decision tree that includes helpful questions about teacher practices and the learning environment to help you decide on how best to work with the teachers you support.

DOMAIN:

Knowledge Progressions and Connections in Numeracy

WHY DOES IT MATTER?

The Knowledge Progressions and Connections domain addresses two related concepts: (1) how teachers sequence and structure their lessons to support learning in the classroom and (2) how teachers link students' experiences, prior knowledge, and learning when presenting new content.

The way that teachers structure and sequence lessons can have an immediate impact on student learning. Research shows that student achievement is maximized when teachers' lessons are structured to include a review, a summary of main points, a gradual increase in difficulty level, and a connection to previous lessons.[3] When teachers state objectives clearly, link them to a specific activity and assessment tasks, and adjust the teaching sequence when needed, students are more successful, are more accountable for their learning, and become more independent.

In addition, having a large body of background knowledge in a given subject makes it easier for students to solve new problems.[4] Effective teachers know that for learning to really "stick," they must make important linkages between new skills and students' prior knowledge, including life experiences and previously taught content. Effective teachers activate students' prior knowledge by reviewing and recalling previously learned content and linking new content to students' life experiences. This supports working memory and enhances learning. The research is clear about the importance of prior knowledge: if such knowledge is ignored, it can put "future learning in jeopardy."[5]

Read more about knowledge progression and connections here.

WHERE TO START?

As mentioned earlier, **not all teachers will start on the same tier of strategies for a given domain**. The decision tree below is designed to help meet the teacher where they are and support them to implement these strategies at their own pace. There are three different boxes for each domain with prompts to help guide decisions as to which strategy teachers – in individual coaching sessions, in communities of practice, or in pre- and in-service training - should try and how to determine when a teacher is ready to implement a new strategy from the next tier. The strategies are color coded: Tier 1 strategies are listed in green, Tier 2 in blue, and Tier 3 in purple. However, it is important to remember and to convey to teachers that the strategies are not meant to be a checklist; it takes time to implement new ideas, and teachers may have to work on a strategy over several days, weeks, or months before feeling confident in using them and before students start benefiting from them.

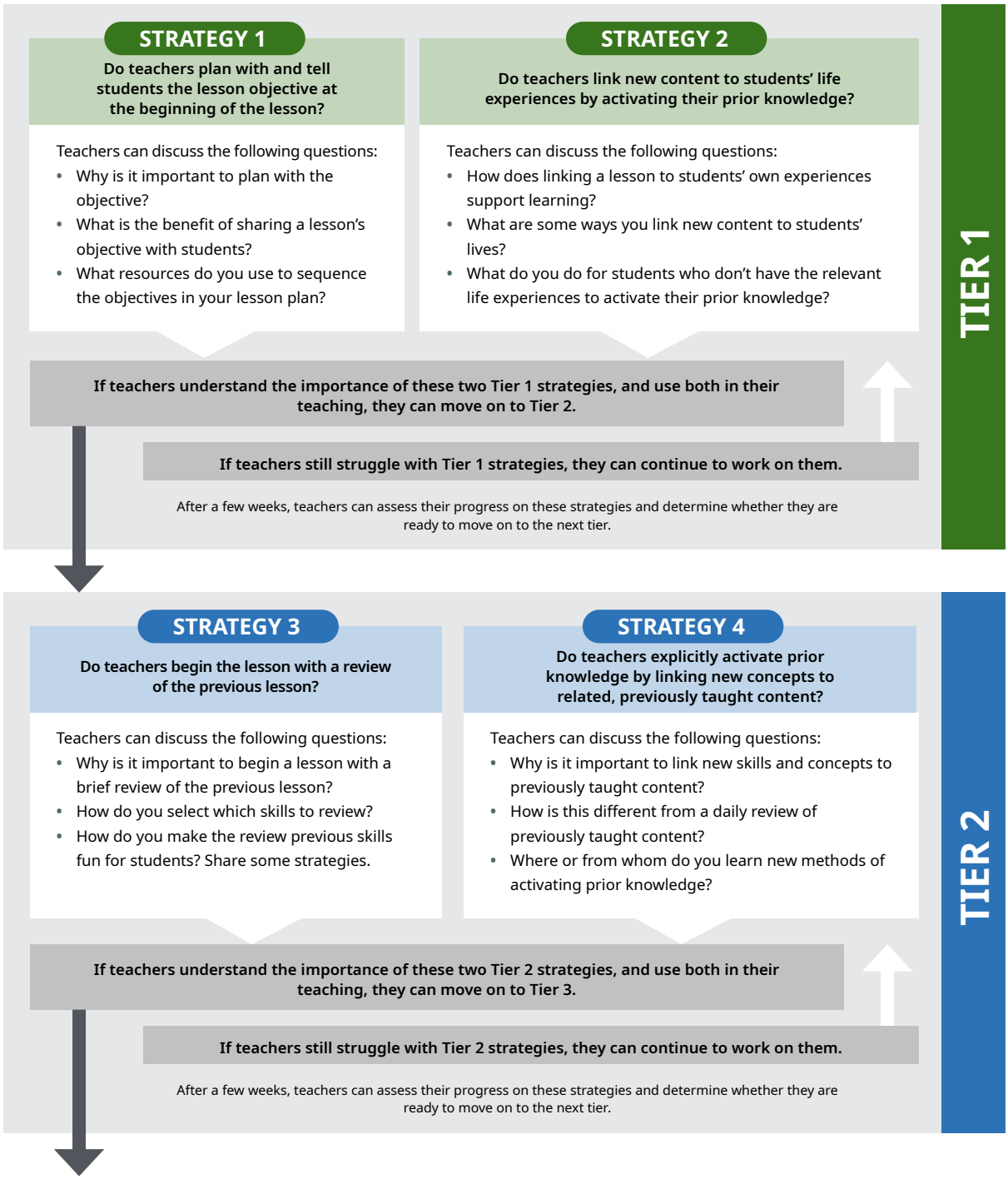
Decision Tree

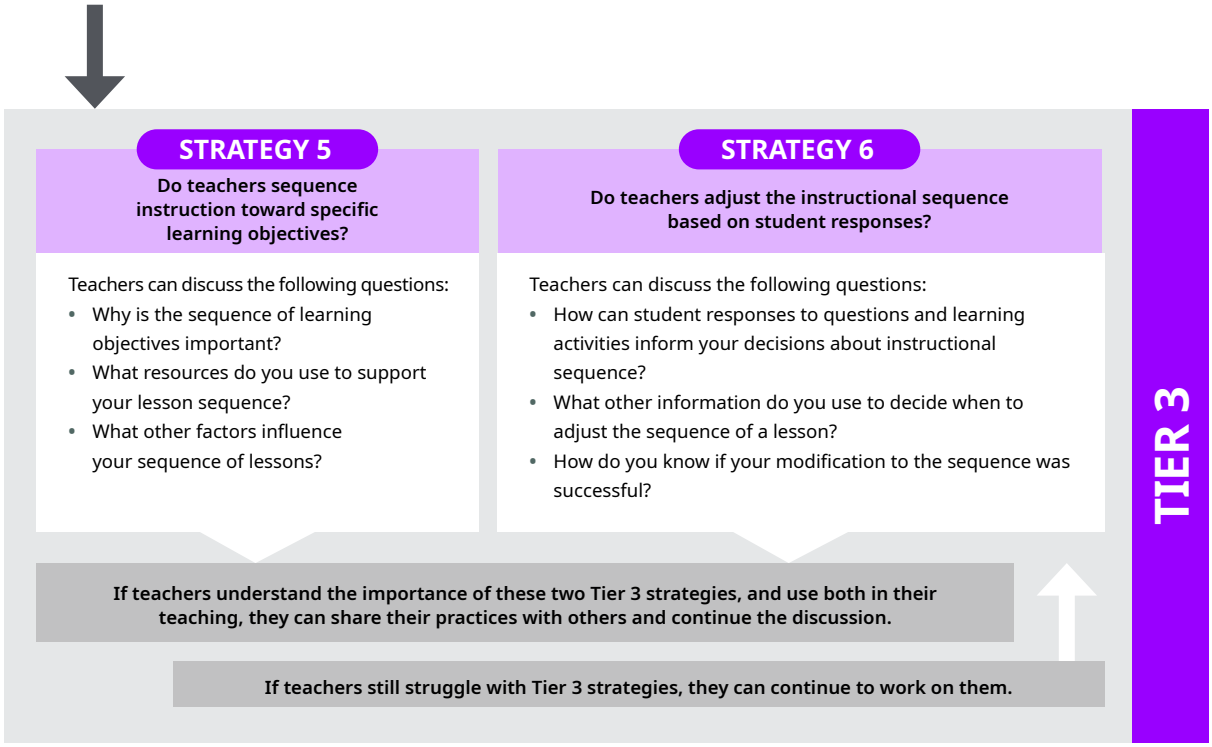
The decision tree below can help teachers identify which strategies to begin with and help those who support their professional development focus their support. Starting with Box 1, teachers can discuss the questions listed to check whether they already implement that strategy. As they proceed through the subsequent boxes in the decision tree, when they reach a strategy that is new to them or they believe needs improvement, teachers work on the strategy noted in that box. The section after the decision tree provides sample activities and approaches for each strategy. Additional strategies offered by teachers should be welcomed.

3 Kyriakides, Christoforou, & Charalambous (2013).
4 Rosenshine (2012).
5 James & Pollard (2011).

Teachers and those who support them can also gauge what type of strategies they use in the classroom by discussing questions such as the following:

- How do you incorporate students' life experiences and background into your lessons?
- When teaching new content, how do you activate students' prior learning?
- Do you review previously taught lesson content before teaching new concepts?
- Do you always follow your lesson plan, or do you make adjustments based on student responses to new content or questions?





STRATEGIES FOR THE MATH CLASSROOM

The strategies below can be used with students across the early primary grades and adapted for the upper grades. Teachers may need to adjust the strategy from the way it is described to work with a specific grade or group of children. The strategies in this mini-guide are written with the classroom teacher in mind, even though, in most cases, coaches or teacher learning facilitators will be the ones sharing the strategies with teachers. Whether it is providing printed handouts or discussing the strategies together, the coach can decide the best way to share them with teachers.

Each of the strategies corresponds to one of three tiers:

Green: TIER 1

Blue: TIER 2

Purple: TIER 3

TIER 1: Teacher plans with and states the lesson objective and links new content to students' background (prior knowledge).

Strategy 1 – Planning with and Stating the Lesson Objective	
Effective teachers plan lessons with a clearly stated lesson objective that is linked to the school's textbook or national curriculum. Stating the objective of the lesson at the start of the class lets children know what to expect, helps them focus their attention on the topic, and helps them become accountable for their learning.	
1.	When developing a lesson plan for a unit on math, begin with a clear objective. The objective of the lesson can be identified or developed by consulting the textbook or the standards, curriculum, and other materials developed by the Ministry of Education.
2.	The objective of the lesson should be clear and simple—and ideally, you should have only one objective per lesson. For example, during a unit on money in which the lesson requires students to "identify the value of coins," the objective might be *By the end of today's lesson, students will be able to identify various coins by name, value, and face.*

3.	Make sure the activity you have planned is linked to the objective. When you start the lesson, share the objective with students. For example, you might say, *In today's lesson, you will learn the names and values of four different coins. They are [hold up each coin as you say its name].*

Strategy 2 – Activating Prior Knowledge: Using Students' Life Experiences

Learning new information becomes easier when it is linked to something familiar. Students bring their own experiences (or prior knowledge) with them to the classroom. Drawing on these experiences and integrating them into lessons helps students build knowledge and learn new information more effectively. Students use their own experiences to make connections with the new content they are learning. Integrating students' life experiences—as in the example below—makes learning more meaningful and relevant to students.

1.	When introducing a new math unit, you can link the content being taught with everyday activities that students might be familiar with. For example, when starting a unit on measurement, you could ask students about their experiences involving measurement when cooking, using questions like these: • *How many of you have gone to the market with one of your parents?* • *What kinds of food do your parents buy at the market?*
2.	Once you have introduced the topic, and students have given you their initial ideas and experiences around measurement related to buying food in the market, you can make linkages with the math lesson: • *When your parent is buying rice [or other food using weight or volume for measuring], how do they know how much to get? [such as using a scale, using a scoop of a certain size to measure, etc.]* • *Do you know how much rice [or, for volume, cooking oil] your parent usually buys?* • *How do you think the seller knows how much to charge?*
3.	As you begin the lesson—for example, discussing units of measurement for weight or volume, or comparing units—refer back to the information that the students have shared about what they have seen or experienced in the market.

TIER 2: Teacher provides a daily review and links content to previous learning.

Strategy 3 – Daily Review

Because math concepts build on one another, a key part of each lesson should be a review of previously taught skills. Starting each new lesson with a review of the skills and concepts that were previously taught will help students recall important information and build connections to new information.

1.	A daily review does not need to be lengthy; three to five minutes are sufficient for reviewing earlier lesson content.
2.	Before launching into the new lesson (using different denominations of coins to "buy" products), take three to five minutes to discuss elements of the previous day's lesson: • *Yesterday we talked about all the different coins we use. Can anyone remind me what they are?* • *You and your partner have a collection of coins. As I select a coin from my basket, work with your partner to find the same coin in your collection. What is this one called? How much is it worth?* • *Working with your partner, arrange the coins in order of their value, starting with the smallest denomination.* • *We have four coins total, starting with the smallest, which is a [name of coin], then [name of coin], then [name of coin], and finally [name of coin].*
3.	During the review, ask questions to probe students' understanding of coins and their value. If it seems like many students are weak on the concept, pause the planned lesson and provide some time to practice that skill.

4.	Reviewing skills and concepts needed in the current lessons is important. If students spend too much time trying to recall old but necessary content, they will not have enough working memory to process new content.

Strategy 4 – Activating Prior Knowledge: Using Previous Learning

Research says that students must connect new knowledge to previous knowledge in order to be successful learners. When students' prior knowledge is sufficiently activated, learning is supported. As a teacher, you play an important role in helping students harness their prior knowledge. The strategy below describes one way to activate prior knowledge and make connections between old and new content.

1.	Design a warm-up activity to link previously learned content to the new skills. For example, before starting a new unit on solving one- and two-step word problems within 50, revisit addition, subtraction, and word problems. Ask students to solve a simple word problem within 20, a concept they will have learned in grade one. You might start your lesson by saying to students: • *Let's practice our addition skills by doing a warm-up activity. This word problem is similar to ones you have done before.* • *Maheen has eight red flowers and seven yellow flowers. How many flowers does she have altogether? Show how you know.* • *Now solve the problem independently in your notebooks.* • Walk around as students work on the problem. Make a note of which students are not able to solve the problem. Offer support to these students immediately. • Give students another problem: *Fauzia has twelve flowers. Five are white and some are yellow. How many are yellow? Show how you know.* • Again, walk around and observe as students work on the problems, giving support to students who are struggling.
2.	In the warm-up activity, you are asking students to solve one- and two-step problems before introducing the more difficult skill of addition and subtraction up to 50. The word problems focus on putting together and taking apart, which are the same skills you will be teaching in the new skill content of solving addition and subtraction word problems within 50.

TIER 3: Teacher purposefully sequences lesson objectives and adjusts the teaching sequence as needed.

Strategy 5 – Purposefully Sequence Lesson Objectives

Well-sequenced lesson objectives support student learning and development. Plans should include tasks and activities that are specifically matched to learning outcomes and use a variety of strategies to meet the learning needs of all students in the classroom. You can sequence lessons using resources such as the textbook and curriculum but should also consider students' skill levels. Having a strong understanding of how skills progress and build on one another also helps you adjust instruction as needed (Strategy 6).

1.	Continuing the example from the previous strategy, let's focus on how you would sequence teaching addition and subtraction word problems within 50. The plan below is an example of how to sequence lessons relating to the topic of addition and subtraction word problems within 50: **Day 1**: Introduce two-step addition word problems. Start with a review of word problems using smaller numbers to help students recall the process. Model the new content, using an example and thinking out loud as you solve the problem. Ask individual questions, call students to the board to solve another example, and assign independent practice questions to all students. Assign paired work (placing stronger students with those who are struggling) in which pairs work on another example of the same type of problem. Observe pairs and determine who needs more support. **Day 2**: Briefly review the content from yesterday, asking a few students to explain the steps involved in solving two-step addition word problems. Assign groups to work on creating their own two-step word problems using addition. Have them share their questions with the class and discuss the questions as a group, making sure the questions make sense. Then assign each group another group's word problem to solve in their groups. Go over each group's responses, asking them to explain and justify their reasoning and answers. **Day 3**: Introduce the same objective—two-step word problems using addition and subtraction—but with larger numbers. Model solving an example and share your thinking aloud as you solve the problem. Ask individual questions, call students to the board to solve another example, and assign independent practice questions to all students. Assign small groups to solve a two-step word problem using addition and subtraction together. Call on groups to share their work and solutions. **Day 4**: Review the content from Day 3, asking students to explain the steps involved in solving two-step addition and subtraction word problems to a partner. Assign pairs to create their own two-step word problems using addition and subtraction. Have pairs first solve their problems and then share their word problems with the class and discuss the questions as a group, making sure the questions make sense. Then assign each pair another pair's word problem to solve. Go over each pair's responses, asking them to explain and justify their reasoning and answers. As always, move around the room, assessing students as they work and offering targeted support as needed. **Day 5**: Begin with a warm-up activity involving mental math. Assign small groups of students to work on activities based on their skill level. For students who were able to solve the word problems with ease, give them additional word problems with larger numbers to solve independently. For the students who need a bit of support with the subtraction word problem, go through an example with them again, modeling and explaining the steps, and provide support as they work on examples. For students who are struggling with the concept of addition and subtraction within 50, do some additional practice problems with them to build and strengthen their understanding of place value and to help them become more fluent at operations.
2.	As stated earlier, observing, asking questions, and using informal assessments to better understand what students have learned is an important aspect of lesson sequencing. There may be pressure to move on, because there are many lessons and units of study to cover in a year, but if students do not master foundational concepts, they will continue to struggle with new material. You may have to adjust your plan based on your observations and assessment data, re-sequence lessons, or adjust your teaching strategy if students do not understand the material well.

Strategy 6 – Adjusting Teaching Sequence

To ensure learning by all students, you will need to use students' responses, reactions, or questions during a lesson to inform the teaching sequence. You may not be able to complete their entire lesson plan in one class or period—this is okay. If it is apparent that students need more time to understand a particular concept, you should adjust the teaching sequence to ensure that students learn the current skills before moving on to new content. It is easier to reteach material than to plan for remediation.

1.	Going back to the previous example, let's imagine that you designed a lesson plan for Day 3 to teach two-step problems involving both addition and subtraction. However, after modeling the new content and going through the steps of solving it, you observed that many students made errors during independent practice. Instead of quickly correcting the errors and moving on to the next step in the lesson, pause and check for understanding and then adjust the sequence of skills you planned to teach.
2.	Check for understanding by doing the following: • Ask a student if they agree or disagree with the other students' explanation of the concept. Probe them to explain why or why not. • Ask a student to summarize the content that you have explained thus far. • Ask a student to explain again in their own words the concept you have taught. These checks for understanding are critical and will help you decide whether students are in the right place to proceed in the sequence of lessons you planned for them or if you need to adjust the sequence to spend additional time teaching the skill again, in a different way, or by providing additional pair work, small-group work, and independent practice.

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